创刊 60 周年专栏

复合材料层合板壳非线性力学的研究进展[1)]

刘人怀[2)]　薛江红[3)]

(暨南大学力学与建筑工程学院应用力学研究所，广州 510632)

(暨南大学重大工程灾害与控制教育部重点实验室，广州 510632)

摘要　复合材料层合板壳是由多种组分材料组合而成. 与单一材料的板壳结构相比，它无明确的材料主方向，各层间材料间断和不连续，具有明显的几何非线性和材料非线性等新的特点. 其失效模式也远比单一材料的情况复杂，具有如基体开裂、脱胶、分层、分层裂纹偏转、多分层以及分层传播等多种模式. 各国学者基于不同的考虑，提出了多种方法研究复合材料层合板壳的失效. 首先, 在简要介绍了层合板壳线性力学基本理论的基础上，重点回顾了层合板壳结构非线性力学几种基本理论发展的过程，主要阐述了经典大挠度非线性理论、一阶剪切变形理论、高阶剪切变形理论、锯齿理论、广义分层理论的理论体系及基本公式，并对几种理论之间的联系和差异进行了总结；其次, 介绍了当前层合结构非线性领域的研究进展，综述了典型复合材料板壳结构的失效机理及优化设计、复合材料板壳结构在复杂环境下的破坏机理、复合材料板壳结构的物理非线性、含脱层纤维增强复合材料板壳结构的破坏机理等各研究热点的最新研究成果；最后, 对该领域未来的研究方向进行了展望.

关键词　复合材料，层合板壳，非线性力学

中图分类号：O34　　**文献标识码**：A　　doi：10.6052/0459-1879-16-253

DEVELOPMENT OF NONLINEAR MECHANICS FOR LAMINATED COMPOSITE PLATES AND SHELLS [1)]

Liu Renhuai[2)]　Xue Jianghong[3)]

(*Institute of Applied Mechanics*，*College of Mechanics and Constructional Engineering*，*Jinan University*，*Guangzhou* 510632，*China*)

(*Key Lab of Disaster Forecast and Control in Engineering of Ministry of Education*，*Jinan University*，*Guangzhou* 510632，*China*)

Abstract　Laminated composite plates and shells are made from a variety of materials. They have quite different mechanical features compared with those made of single material, such as uncertain principal direction of the material, discontinuity of material between layers, highly geometrical and material nonlinearity, etc. Their failure modes include matrix cracking, debonding, delamination, crack deflection, multi delamination and delamination propagation, which are much more complex than those of single material. Based on different considerations, various methods have been proposed by scholars from different countries to study the failure of laminated composite plates and shells. This paper summarizes the fundamental theory of linear mechanics and reviews the development of nonlinear theories for laminated plates and shells. In particularly, theoretical systems and basic formulas are expatiated for the classical nonlinear theory of large deformation, the first order shear deformation theory, the high order shear deformation theory, the zig-zag theory, and the

2016–09–08 收稿，2016–12–02 录用, 2016–12–02 网络版发表.

1) 国家自然科学基金资助项目 (11032005).

2) 刘人怀，院士，教授，主要从事板壳力学和复合材料非线性结构力学研究. E-mail: lrh@jnu.edu.cn

3) 薛江红，教授，主要从事不完善复合材料板壳结构的失效机理研究. E-mail: txuej@jnu.edu.cn

引用格式：刘人怀，薛江红. 复合材料层合板壳非线性力学的研究进展，力学学报, 2017, 49(3): 487-506

Liu Renhuai, Xue Jianghong. Development of nonlinear mechanics for laminated composite plates and shells. *Chinese Journal of Theoretical and Applied Mechanics*, 2017, 49(3): 487-506

layer-wise theory. The relevance and differences among these theories are stated. Current research progress in the field of nonlinear mechanics for laminated composite plates and shells are overviewed and the latest achievements are introduced in research hotspots regarding the failure mechanism and optimization design of typical laminated composite plates and shells, the failure mechanism of laminated composite plates and shells in complex environments, material nonlinearity of composite plates and shells, failure mechanism of fiber reinforced delaminated composite plates and shells, and so on. Based on the review, prospects for future research in the area of nonlinear mechanics of laminated composite plates and shells are proposed.

Key words composite materials, laminated plates and shells, nonlinear mechanics

引 言

随着世界各国在航天、航空、石油化工、机械工程、海洋工程、交通运输、能源领域的不断发展，传统的单一材料已不能满足日益增加的对材料高强度、高刚度、高断裂韧性和低比重等综合优良性能的要求, 寻求先进复合材料已成为材料技术发展过程中的一个关键. 我国政府对先进材料的研究和开发大力支持, 在 “863 计划” 与 “973 计划” 中, 先进材料技术分别为其中的 7 个与 6 个重点领域之一. 近期实施的重大基础研究攀登计划中, 30 个重大课题有 7 个与材料有直接关系. 国家自然科学基金委资助的研究课题，与材料有关的约占 1/4. 综上所述, 先进材料技术已成为我国当前经济建设中的一项关键技术, 因此针对先进复合材料广泛开展的研究工作具有重要的意义.

关于复合材料，广义上来讲，它是由两种或者两种以上具有不同性质的材料，经过物理或化学的方法复合而形成的一种具有新性能的多相固体材料. 层合板壳结构则是将多层复合材料的单层板壳粘合在一起组成的结构体系. 由于复合材料层合板壳是由多种组分材料组合而成, 与单一材料的板壳结构相比，无明确的材料主方向, 各层间材料间断和不连续, 具有明显的几何非线性和材料非线性等新的特点. 其失效也具有如基体开裂、脱胶、分层、分层裂纹偏转、多分层以及分层传播等多种模式，因此需要建立与层合板壳相适应的几何、本构模型来描述层合板壳的结构整体性能和力学性能. 本文将从结构非线性力学的角度对复合材料层合板壳的研究情况进行回顾和综述.

1 复合材料层合板壳非线性理论

在板壳弯曲理论中 [1-2]，当层合板受到一般载荷作用时，总可以把每一个载荷分解为两个分载荷：一个是作用在结构中面之内的纵向载荷，另一个是垂直于中面的横向载荷. 纵向载荷和横向载荷所引起的应力、应变和位移分别按平面应力问题和板壳弯曲问题进行计算. 层合板的平衡微分方程为

$$\left.\begin{aligned}&\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\&\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0\\&\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial xy}+\frac{\partial^2 M_y}{\partial y^2}+\\&\qquad N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial xy}+N_y\frac{\partial^2 w}{\partial y^2}+q=0\end{aligned}\right\}\tag{1}$$

其中，M_x, M_y, M_{xy} 为层合板横向的弯曲内力，N_x, N_y, N_{xy} 为作用于中面内的薄膜内力，q 为横向载荷.

根据层合板的应力应变关系和有关假设 [3]，层合板的本构方程为

$$\left.\begin{aligned}&\begin{bmatrix}N_x\\N_y\\N_{xy}\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}&A_{16}\\A_{12}&A_{22}&A_{26}\\A_{16}&A_{26}&A_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\\varepsilon_y\\\varepsilon_{xy}\end{bmatrix}+\\&\qquad\begin{bmatrix}B_{11}&B_{12}&B_{16}\\B_{12}&B_{22}&B_{26}\\B_{16}&B_{26}&B_{66}\end{bmatrix}\begin{bmatrix}\chi_x\\\chi_y\\\chi_{xy}\end{bmatrix}\\&\begin{bmatrix}M_x\\M_y\\M_{xy}\end{bmatrix}=\begin{bmatrix}B_{11}&B_{12}&B_{16}\\B_{12}&B_{22}&B_{26}\\B_{16}&B_{26}&B_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\\varepsilon_y\\\varepsilon_{xy}\end{bmatrix}+\\&\qquad\begin{bmatrix}D_{11}&D_{12}&D_{16}\\D_{12}&D_{22}&D_{26}\\D_{16}&D_{26}&D_{66}\end{bmatrix}\begin{bmatrix}\chi_x\\\chi_y\\\chi_{xy}\end{bmatrix}\end{aligned}\right\}\tag{2}$$

其中，$\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$ 是层合板的中面应变，$\chi_x, \chi_y, \chi_{xy}$ 是其中面曲率、扭曲率，A_{ij} 为层合板的拉伸刚度，表示的是薄膜内力与中面应变的刚度关系，D_{ij} 为弯曲

刚度，表示的是弯曲内力矩与曲率及扭曲率有关的刚度系数；B_{ij} 为耦合刚度，表示弯曲、拉压之间的耦合关系. 若令 E_1，E_2，μ_{12}，μ_{21} 为正交各向异性单层板主方向的材料常数，$Q_{ij}^{(k)}$ 和 θ 分别为第 k 层的弹性刚度和该层坐标系与材料主方向的夹角，则有

$$\left.\begin{aligned} A_{ij} &= \sum_{k=1}^{n} Q_{ij}^{(k)}(z_k - z_{k-1}) \\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{n} Q_{ij}^{(k)}(z_k^2 - z_{k-1}^2) \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{n} Q_{ij}^{(k)}(z_k^3 - z_{k-1}^3) \end{aligned}\right\} \tag{3}$$

$$\begin{bmatrix} Q_{11}^{(k)} \\ Q_{12}^{(k)} \\ Q_{22}^{(k)} \\ Q_{16}^{(k)} \\ Q_{26}^{(k)} \\ Q_{66}^{(k)} \end{bmatrix} = \begin{bmatrix} c^4 & 2c^2s^2 & s^4 & 4c^2s^2 \\ c^2s^2 & c^4+s^4 & c^2s^2 & -4c^2s^2 \\ s^4 & 2c^2s^2 & c^4 & 4c^2s^2 \\ c^3s & cs^3-c^3s & -cs^3 & -2cs(c^2-s^2) \\ cs^3 & c^3s-cs^3 & -c^3s & 2cs(c^2-s^2) \\ c^2s^2 & -2c^2s^2 & c^2s^2 & (c^2-s^2)^2 \end{bmatrix} \begin{bmatrix} Q_{11} \\ Q_{12} \\ Q_{22} \\ Q_{66} \end{bmatrix} \tag{4}$$

$$\left.\begin{aligned} &Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}},\quad Q_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}} \\ &Q_{12} = \mu_{12}Q_{22} = \mu_{21}Q_{11},\quad Q_{66} = G_{12} \\ &E_2 = \mu_{21}E_1,\quad c = \cos\theta,\quad s = \sin\theta \end{aligned}\right\} \tag{5}$$

在纵向载荷和横向载荷的作用下，若板内产生的应变量和转动分量都远远小于 1，且应变量与转动分量为等量级的微小量时，由线性理论知

$$\left.\begin{aligned} &\varepsilon_x = \frac{\partial u}{\partial x},\quad \varepsilon_y = \frac{\partial v}{\partial x},\quad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ &\chi_x = -\frac{\partial^2 w}{\partial x^2},\quad \chi_y = -\frac{\partial^2 w}{\partial y^2},\quad \chi_{xy} = -\frac{\partial^2 w}{\partial x \partial y} \end{aligned}\right\} \tag{6}$$

当作用在复合材料层合板上的外载荷较大，所引起的面外变形与厚度之比不再是小量时，面内应变就必须考虑，此时薄膜力在弯曲平衡时起了很大的作用，形成几何非线性问题. 1910 年，von Kármán[4] 首先给出了平板大挠度非线性方程，此方程中考虑了应变分量的二阶小项及薄膜应力在变形后位形上对平衡方程的影响，从此奠定了板壳几何非线性分析的基础 [5-6]. 现有的复合材料层合板壳的非线性理论是从上述理论发展起来的，可分为以下 5 种：经典大挠度弯曲理论，一阶剪切变形理论，高阶剪切变形理论，锯齿理论和广义分层理论.

1.1 经典大挠度弯曲理论

经典大挠度弯曲理论是依据基尔霍夫 (Kirchhoff) 直法线假设，忽略了横向剪切变形而建立的. 根据 Kirchhoff 假定，变形前垂直于中面的直线在变形后仍然垂直于中面，在层合板内距中面为 z 的点，其位移沿厚度做线性变化，即

$$\left.\begin{aligned} u^{(z)} &= u(x,y) - z\frac{\partial w}{\partial x} \\ v^{(z)} &= v(x,y) - z\frac{\partial w}{\partial y} \\ w^{(z)} &= w(x,y) \end{aligned}\right\} \tag{7}$$

根据基尔霍夫的应变假设在板内任一点处有 $\varepsilon_{zx} = \varepsilon_{zy} = \varepsilon_z = 0$. 由上式得结构内任意点的应变表达式

$$\left.\begin{aligned} \varepsilon_x^{(z)} &= \varepsilon_x - z\frac{\partial^2 w}{\partial x^2} \\ \varepsilon_y^{(z)} &= \varepsilon_y - z\frac{\partial^2 w}{\partial y^2} \\ \gamma_{xy}^{(z)} &= \gamma_{xy} - z\frac{\partial^2 w}{\partial x \partial y} \end{aligned}\right\} \tag{8}$$

其中，ε_{ij} 为板中面的应变.

在小应变 ($\varepsilon_x, \varepsilon_y, \varepsilon_{xy} \ll 1$)、大转动的非线性问题中，由于板的挠度 $w \gg u, v$，根据中等大挠度的 von Kármán 非线性几何公式，层合板的中面应变、曲率改变量的表达式为

$$\left.\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, & \chi_x &= -\frac{\partial^2 w}{\partial x^2} \\ \varepsilon_y &= \frac{\partial v}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, & \chi_y &= -\frac{\partial^2 w}{\partial y^2} \\ \varepsilon_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, & \chi_{xy} &= -\frac{\partial^2 w}{\partial x \partial y} \end{aligned}\right\} \tag{9}$$

在 20 世纪 60 年代中后期 Stavsky[7]，Whitney 和 Leissa[8] 最先建立了基于 von Kármán 非线性几何公式的适用于各向异性材料的非线性理论. Chia 等 [9-17] 采用摄动法和广义多重 Fourier 级数方法，对斜交层合板、非对称各向异性层合板在不同边界条件下的非线性弯曲问题进行了研究. 然而使用幂级数解法和其他方法去处理层合壳体非线性问题，异常困难，解的精确度常常不能满足要求. 为此，20 世纪 90 年代初，叶开沅等在其前期板壳非线性研究成果基础上 [18-39]，创造性地将修正迭代法引入到层合结构的非线性问题分析中，率先研究了层合矩形板的非线性弯曲 [40]，以及极难求解的层合扁球壳和层合扁锥壳的非线性稳定 [41-42] 等问题，解决了复

合材料层合板壳领域的一大难题. 他们不仅求解了非线性控制微分方程，获得了临界载荷的解析解，而且他们的解法结合了摄动法和逐次逼近法的优点, 不仅程序简单、计算量小, 而且收敛快、所获解析解的精度高.

此外，Turvey 和 Wittrick[43] 采用动力松弛法研究了简支斜交层合板的大挠度和后屈曲问题. Zaghloul 和 Kennedy[44] 对对称正交或斜交层合板在四边夹紧固定或简支两种边界条件下的大挠度问题进行了实验研究, 并给出了相应的有限差分解. Chandra[45] 使用伽辽金法分析了一个四边固定反对称层合板的非线性弯曲问题. 周次青 [46-48] 采用奇异摄动法讨论了多种边界条件下各向异性层合板的非线性弯曲问题, 导出了挠度和应力函数的 N 阶一致有效渐近展开式.

1.2 一阶剪切变形理论

层合板/壳的经典大挠度弯曲理论是在经典层合板理论基础上，考虑了层合板中面的薄膜变形以及面外的横向大挠度对薄膜变形的影响. 但经典层合板壳理论却忽略了横向剪切变形，只考虑了横向剪力的作用. 由于层合板/壳是由很多各向异性的叠片按一定规律堆砌而成，其等效横向剪切模量远小于面内纤维方向的等效弹性模量，因此层合板对横向剪切变形很敏感. 基于此考虑产生了一阶剪切变形理论. 在一阶变形理论中，位移场取为

$$\left.\begin{aligned} u^{(z)} &= u_0(x,y) - z\psi_x \\ v^{(z)} &= v_0(x,y) - z\psi_y \\ w^{(z)} &= w_0(x,y) \end{aligned}\right\} \tag{10}$$

其中, u_0, v_0 和 w_0 为板中面的变形，ψ_x 和 ψ_y 为板内任一点处法线对板中面的转角.

一阶剪切变形理论基于 Reissner–Mindlin 中厚板理论 [49-50]，考虑了横向剪切变形的影响，最早由 Stavsky[51] 针对各向同性层合板提出，之后 Yang 等 [52] 将该理论推广到各向异性板，在二十世纪七八十年代被 Ambartsumyan[53]，Bert[54] 和 Whitney[55] 引入到各向异性板和层合板中. Dong 和 Tso[56] 建立了层合圆柱壳的一阶剪切变形理论. 之后，Reddy 和他的合作者们 [57-59] 将 von Kármán 非线性几何方程引入到 Reissner –Mindlin 剪切变形理论中，考虑了小应变、大转动的几何非线性，建立了各向异性板考虑中面薄膜影响的非线性一阶剪切变形理论，并基于该理论应用有限元算法求解问题 [60-61]. 由于模型简单，变量少，理论值较符合实验值，一阶剪切变形理论得到了广泛的应用. Iu 等 [62-66] 研究了层合圆柱壳、圆锥壳的后屈曲和非线性振动，Fares 等 [67-70] 运用 Reissner 的混合变分原理，提出了一种考虑热效应的精细一阶剪切变形理论，使得应力在上下边界处得以连续. Whitney 和 Pagano[71] 分析了对称和反对称铺设的正交及斜交矩形板受正弦分布载荷的弯曲与自由振动. Fortier 和 Rosettos[72] 以及 Sinha 和 Rath [73] 分别对非对称斜交铺设的矩形厚板的自由振动和屈曲进行了研究. Bert 和 Chen[74] 给出了反对称斜交铺设的简支矩形板自由振动的近似解.

在层合板壳计及一阶剪切变形非线性理论领域中，刘人怀等创造性地将一阶剪切变形理论引入到各种复合材料层合薄壁板壳结构中，结合修正迭代法，率先涉及复杂的复合材料层合旋转壳体领域, 建立了计及横向剪切影响的对称层合扁球壳和扁锥壳的非线性稳定和振动理论, 系统地分析了对称层合圆柱正交异性扁球壳的非线性稳定问题 [75]、非线性强迫振动 [76] 与动态屈曲 [77]、层合圆板的大挠度弯曲 [78] 与受迫振动 [79]、椭圆板的大挠度弯曲 [80]、开顶扁球壳的非线性稳定性 [81-82]、非线性强迫振动 [83] 和动态屈曲 [84]、扁锥壳的非线性屈曲 [85]. 这些问题的研究结果可直接用于工程设计，刘人怀等得出了重要的结论：若使用不计及横向剪切影响的经典理论进行工程设计, 那将是保守的. 同时刘人怀和王志伟还率先研究了考虑面层和芯层的横向剪切变形的复合材料面层夹层板壳的非线性一致有效理论 [86-87]，给出了复合材料面层夹层矩形板的非线性弯曲 [88] 和夹层扁壳的非线性弯曲与屈曲的解答 [88-90]. 接着，徐加初等 [91] 研究了具有夹层的正交异性复合材料圆锥壳体的非线性轴对称屈曲问题.

由于一阶剪切变形理论假设恒定的横向剪切应力，不满足在自由边界无面力的边界条件. 为了克服这个问题，提出了各种解决的办法. 一种办法是引入一个剪切校正因子，文献 [92-97] 中给出了几种情况下的剪切校正因子；另一种办法是对一阶剪切变形理论进行细化.

1.3 高阶剪切变形理论

一阶剪切变形理论在预测层合结构的整体响应，即：挠度，固有频率和屈曲载荷方面提供了良好的结果，但在研究沿厚度方向的应力响应方面，一阶剪切变形理论给出的结果都远远偏离了结构的实际

情况. 为了克服一阶剪切形变理论的局限，出现了各种高阶剪切变形理论. 在二阶和三阶变形理论中，位移分量被展开成层合板厚度的高阶多项式 (二次和三次)，其中，以 Reddy [98-99] 在 1984 年提出的三阶剪切理论影响最为广泛. 在 Reddy 的三阶剪切变形理论中包含了更多的位移函数，其位移表达式具有如下的形式

$$\left.\begin{aligned} u(x,y,z) &= u_0(x,y) + z\psi_x(x,y) + z^2\phi_x(x,y) + \\ &\quad z^3\theta_x(x,y) \\ v(x,y,z) &= u_0(x,y) + z\psi_y(x,y) + z^2\phi_y(x,y) + \\ &\quad z^3\theta_y(x,y) \\ w(x,y) &= w_0(x,y) \end{aligned}\right\} \tag{11}$$

其中，u_0, v_0, w_0, ψ_x, ψ_y, ϕ_x, ϕ_y, θ_x, θ_y 是坐标 (x,y) 的未知函数，且 u_0, v_0, w_0, ψ_x, ψ_y 的含义与一阶变形理论一致. 在此基础上，Reddy[100] 进一步引入 von Kármán 非线性几何方程，建立了非线性的高阶剪切形变理论，Reddy 和 Liu[101] 建立了层合壳的高级剪切变形理论. 根据 Reddy 理论得到的横向剪切应变沿厚度方向呈抛物线分布，因此该理论不需要剪切修正，被大量用来计算层合板壳的挠度、自然频率和屈曲载荷 [102-109]. 除了 Reddy 的理论，Kant 和 Swaminathan[110-112] 建立了考虑横向应变的三阶理论，其中面内位移和横向位移都为横向坐标的三次多项式, 并分析了复合材料层合板/夹层板自由振动与弯曲问题. Barut 等 [113] 发展了三阶理论分析多层夹层板弯曲问题, 其中面内位移为三次多项式而横向位移为二次多项式.

与国际研究同步，国内也开展对高阶剪切变形理论的研究. Reissner-Mindlin 型板理论不能满足在板的上、下表面横向剪切应力为零的条件，而三维理论用于分析实际问题相当难以操作. 为此，Liu 和 He[114] 应用虚位移原理，创新地提出了一个计及高阶影响的复合材料层合板的非线性弯曲问题的简化理论. 通过将板的总挠度分解为一个由弯曲而产生的挠度和一个由剪切而产生的挠度，进而建立了较为简单的控制方程. 他们的理论首次说明了横向剪切应变在厚度上的抛物线变化的原因，并使得控制方程的求解更简单，所得到的结果与 Reddy 的三维理论 [115] 所得的结果相当一致. 沈惠申运用 Reddy 的高阶剪切理论研究了复合材料层合板热后屈曲 [116] 和复合材料层合剪切圆柱曲板在侧压作用下的后屈曲 [117]. 冯世宁和陈浩然 [118] 基于 Reddy 提出的层合板高阶剪切变形简化理论对复合材料层合板的非线性动力稳定性问题进行了研究. 张雨和向锦武 [119] 对复合材料层合厚圆柱壳高阶理论进行改进，提出了一个改进的精化高阶理论. 文献 [120-127] 给出了其他的二阶和三阶的剪切变形理论.

Matsunaga[124-129] 在研究二阶和三阶理论的基础上提出了的九阶理论，Reissner[130] 给出了十二阶理论. 在上述的各种高阶理论中，都是将位移函数展成厚度坐标的多项式系列. Ambartsumian 在文献 [131] 中综述了各种高阶剪切理论，并首次提出横向剪切应变形状函数. 此外，还提出了一些新的高阶理论，在这些高阶理论中横向剪切应变形状函数不是取为多项式函数，而是取为其他的函数，如 Kaczkowoski[132], Panc[133] 和 Reissner[49], Levinson[134], Reddy 和 Liu[101] 等学者的理论. 在 Ghugal 和 Sayyad[135] 提出的一种高阶剪切理论里，其剪切应变形状函数为三角函数，El Meiche 等 [136] 引入一个双曲剪切变形函数作为剪切变形函数，Matsunaga[137] 将位移场展成厚度坐标的幂级数，Karama 等 [138-140] 和 Aydogdu[141-142] 运用了指数函数，Neves 等 [143] 采用了双曲函数和正弦函数.

1.4 锯齿理论

现有的大多数层合板理论都建立在位移假设基础上，这要归因于基于位移的理论在发展层合板计算模型时表现出的简单性. 按照位移假设方式的不同，这些理论可以归结为两类：一类是将非均匀的各向异性层合板等效为均匀各向异性单层板，并将板的位移沿整个厚度假设成厚度方向坐标的连续光滑函数. 这方面的理论有基于面内位移沿厚度线性分布假设的一阶理论，以及基于面内位移沿厚度非线性分布假设的高阶理论. 显然，由于这类理论中应变在层合板层间连续，因而横向剪应力在层间间断. 另一类理论则采用逐层假设位移的方法，预先满足层间位移和横向剪切应力连续的条件，即锯齿理论.

锯齿理论最早由 Murakami[144] 提出. 为了模拟面内位移沿厚度方向呈之字形的分布情况，Murakami 在面内位移场上添加一个锯齿函数. 之后，通过在勒让德多项式基础上添加一个锯齿函数，Murakami 和 Toledan[145] 提出了一个改进的锯齿理论，并据此分析了层合板的圆柱弯曲问题. 锯齿理论比单纯提高位移场阶数更为有效，不仅计算精度比高阶理论高，而且位移的未知量数目也比高阶理论少，因此被

用来分析层合板壳的静动力性能. Carrera[146] 将 Murakami 的锯齿函数 (Murakami zig-zag function, MZZF) 引入一阶和高阶剪切位移场，详细研究了简支层合板壳的静力、动力和热载荷响应，并分析了 MZZF 的有效性和局限性. Bhaskar 和 Varadan[147] 运用 MZZF 分析了正交层合旋转壳的弯曲问题，并建立了基于 Murakami 的锯齿函数的高阶理论. Vidal 和 Polit[148] 把 MZZF 添加到正弦位移场, 并研究了复合材料层合梁弯曲问题. Ferreira 等 [149] 和 Rodrigues 等 [150] 提出了一种变异的 MZZF，并运用径向基函数配置法研究了层合板的静力问题、自由振动和屈曲问题. 文献 [151-155] 介绍了基于 Murakami 的锯齿理论的各种有限元分析.

Murakami 的锯齿理论虽然满足层间位移连续的条件 (C^0 连续)，但不满足层间横向剪切应力的连续条件，因此不能用来确定层间的应力分布. 为了克服这一问题，Di Sciuva 等 [156-158] 在假设层合板面内位移沿厚度呈折线分布，即面内位移沿厚向呈锯齿形分布的基础上，提出了预先满足层间应力连续的锯齿理论，并进一步推广此理论, 用之分析复合材料板壳. 然而，Di Sciuva 的理论中横向剪应力在厚度方向上是均匀的，不满足上下表面处的横向剪应力协调的条件. 1992 年，Cho 等 [159-161] 在之字形的面内线性位移基础上叠加一个三次的位移场，提出了适于分析复合材料层合结构的锯齿理论. 在此锯齿理论中，层合板壳结构中第 k 层任一点的位移场可以统一地表示为

$$\left.\begin{aligned} u^k(x,y,z) &= u_0(x,y) + z\psi_x(x,y) + z^2\varphi_x(x,y) + \\ &\quad z^3\theta_x(x,y) + \sum_{j=1}^{k-1} S_x^j\left(z-z_j\right)H\left(z-z_j\right) \\ v^k(x,y,z) &= v_0(x,y) + z\psi_y(x,y) + z^2\varphi_y(x,y) + \\ &\quad z^3\theta_y(x,y) + \sum_{j=1}^{k-1} S_y^j\left(z-z_j\right)H\left(z-z_j\right) \\ w^k(x,y,z) &= w_0(x,y) + zw_1(x,y) + z^2w_2(x,y) \end{aligned}\right\} \tag{12}$$

式中，$S_x^j(z-z_i)$ 和 $S_y^j(z-z_i)$ 为第 j 层的位移增量，$H(z-z_i)$ 为阶跃函数. 由层间横向剪应力的协调性和表面剪应力为零的条件，可以建立 S_x^j 与 S_x^{j-1}, S_y^j 与 S_y^{j-1} 的递推关系，并推导出 ψ_x, ψ_y 与 w_0 以及 ϕ_x, ϕ_y 与 w_1, w_2 关系表达式.

Cho 的锯齿理论适合于分析层合板壳由纯机械载荷引起的弯曲、屈曲及动力性能. 对于受机械载荷与热载荷共同作用下的层合板/壳结构的静动力问题，横向的正应变就不能忽略，因为在此情况下，面外的热变形与面内热变形几乎相当 [162]，为此，Cho 等 [163-164] 提出了考虑横向正应变的三阶锯齿理论. 该理论被用来分析含多个分层的层合壳的弹性屈曲，以及黏弹性层合板的力学性能 [165] 和在不同边界情况下层合板的弯曲性能 [166]. 之后，Icardi[167-168] 采用面内位移和面外位移沿厚度分别呈三次和四次分布，提出了适合厚壳的修正锯齿理论.

与此同时, 锯齿理论在国内的研究也取得了令人瞩目的成果. 1994 年，何陵辉等 [169] 发展了一种高阶锯齿理论，开创了锯齿理论在国内研究的新局面. 国内外曾有过许多层合复合材料板壳的力学模型，但这些模型或基于分层的运动学假设，使未知量数目随层数急剧增加, 因而计算极为复杂; 或引入整个厚度上的光滑位移假设，虽使未知量数与层数无关却不能满足层间应力协调条件. 何陵辉等提出了逐层光滑的连续位移表达式，建立了任意形状层合壳体的线性和非线性一般理论，不但使表面和层间应力协调条件自动满足，而且未知量数目与层数无关 (5 个). 通过一些特例的计算表明，他们提出模型的预言结果与相关精确解无论在局部还是在整体力学响应上都十分吻合. 因此，在兼顾计算的简单性和结果的精确性方面，他们提出的模型优于国内外同类工作. 之后，舒小平 [170] 提出了一种简单高阶剪切变形层板理论, 其位移场满足层间位移和横向剪应力的连续性，并将该理论推广用于分析层合壳的热弹性动力响应 [171]. 白瑞祥和陈浩然 [172] 运用 zig-zag 模型和 Mindlin 一阶剪切变形板理论, 考虑面板的横向剪切变形和芯体的面内刚度对夹层板力学性能的影响，建立了一种分层模型和多标量损伤模型. 任晓辉和陈万吉 [173] 发展了 C^0 型锯齿理论，通过虚位移原理推导出在热载荷作用下复合材料梁的平衡方程，并给出了简支复合材料层合梁解析解.

锯齿理论虽然能使层间的横向剪应力连续，但依据该理论的有限元模型需要满足位移的 C^1 连续条件. 而要在任意四边形和三角形单元的接口处施加 C^1 连续条件极难实现，且其单元刚度矩阵的计算极其耗时. 为了获得准确的层间应力还需要借助三维的平衡方程 [174-178]. 其次锯齿理论中没有考虑横向正应变的影响，不能精确地确定在材料间断处，如孔边、开孔处、上下界面、脱层尖端等处的应力情况. 锯齿理论的另一个限制是横向剪切应力在层合

板的边缘处不能满足自由边界条件.

1.5 广义分层理论

对于较厚的层合梁、板和壳等结构，其层间剪切和拉、压应力呈三维应力状态，采用等效单层板的板壳理论分析，已不能满足精度要求. 为此，Reddy[179] 提出了广义分层理论. 该理论采取在板的厚度方向取线性插值函数来描述每个数值层内位移沿厚度方向的变化规律. 在线性理论基础上，Reddy 等 [180-181] 又考虑 von Kármán 几何关系，建立了非线性的分层理论，并提出了考虑脱层的层合板分层理论 [182]，其位移场取为

$$\left.\begin{aligned} u(x,y,z) &= u_0(x,y) + \sum_{i=1}^{N} U_i(x,y)\phi_x^i(z) \\ v(x,y,z) &= v_0(x,y) + \sum_{i=1}^{N} V_i(x,y)\phi_y^i(z) \\ w(x,y,z) &= w_0(x,y) + \sum_{i=1}^{N} W_i(x,y)\phi_z^i(z) \end{aligned}\right\} \tag{13}$$

其中，u_0, v_0, w_0 为板中面的变形，N 为沿厚度方向层合板的子板层数，U_i, V_i, W_i 为第 i 个子层的位移分量，$\phi_x^i, \phi_y^i, \phi_z^i$ 是厚度坐标的分段连续插值函数，由相邻的两个子板决定. 分层理论将层合板沿厚度方向分成若干子层，每个子层视为一个由相同材料构成的单层板，可由多个铺设层构成，总的子层板数可以小于层合板的铺设层数. 位移在厚度方向是 C^0 连续的，但其导数在层间不连续，通过运用三维的本构方程保证了横向剪切应力在层间连续，并且上下表面的协调性也得到满足.

由于 Reddy 的分层理论中，面内位移为沿厚度线性分布的，因此所得的剪应力和剪应变在各层内为均匀分布，而非抛物线分布. 为了克服这一问题，出现了各种高阶分层理论，即是在线性的分层位移基础上再叠加高阶位移场. Di Sciuva 和 Gherlone [183] 基于 Hermite 分层位移场，构造了一种满足 C^1 连续的，含贯穿厚度损伤的有限元梁模型，并将该模型推广到含损伤的层合板 [184]，建立了满足 C^0 连续的八节点板单元 [185]. Pai 和 Palazotto [186] 提出了一种高阶分层理论，在他们的理论中，每一层面内位移沿厚度呈二次和三次分布，而面外位移沿厚度呈二次分布. Carrera[187-189] 发展了一种预先满足层间位移，横向剪应力和正应力连续条件的混合分层理论，并建立了有关的有限元模型. 在其混合分层理论中，每一层的位移场和应力场被展成 N 阶的勒让德多项式. 而 Carrera 和 Demasi[190] 以及 Icardi[191] 则运用混合分层理论分析了夹层板的力学性能.

由于分层理论能够描述层合板壳内每一层的独自变形情况，因此被广泛用来分析层合结构的静动力问题. Marjanovic 和 Vuksanovic[192] 进一步改进了分层模型，以考虑层合板壳内脱层的影响，并运用改进的模型计算含脱层层合板的自然频率、振型和屈曲载荷. Alnefaie[193] 运用分层有限元模型，对含层间损伤层合板的基本动力特性进行了分析. Basar 等 [194-195] 开发了一系列的多层壳单元模型来计算层间应力，并用分层理论的壳模型来研究层合结构的自由振动响应. 为了降低计算成本，Botello 等 [196] 采用三角形单元导出基于分层理论的有限元模型，并提出了一种子结构技术以消除刚度矩阵装配过程中的面内自由度. Ghoshal 等 [197-198] 运用一种精细的分层理论分析了脱层层合板的线性和非线性振动，在他们的分析中考虑了脱层处的接触效应. 更多关于分层理论的成果，可参阅文献 [199-204].

分层理论是一种准三维理论，其特点是未知变量个数依赖于层合板层数，因此，当层合板层数增加时，其计算量将大大增加. 其次，分层理论不能预先满足层间应力连续条件，需要采用三维平衡后处理方法来计算层间应力.

总的来说，层合板壳层间界面的材料不连续性会导致层合板壳内部位移和应力分布的复杂性. 文献 [205-211] 综述了最近几十年剪切变形理论的发展情况.

2 复合材料层合板壳非线性力学性能

对纤维增强复合材料层合板壳结构在弹性变形条件下的非线性力学分析，现有的研究已经由对单一结构、简单加载等问题的研究转为了对复合结构、在复杂加载环境下的性能研究，由线性材料推广到考虑断裂、蠕变等非线性材料性质，由完善结构到含缺陷的不完善结构.

2.1 典型复合材料板壳结构的失效机理及优化设计

刘人怀与合作者 [6,7,40-42,75-85,87-91,169] 最早在我国开展复合材料层合板壳结构的一系列非线性力学问题研究. 他们的研究涉及工程结构中最重要的 5 类板壳：层合圆板、层合矩形板、层合椭圆板、层合扁球壳和层合扁锥壳，同时还研究了复合材料夹层板壳，针对这些结构在典型载荷作用下的非线性弯曲、稳定和振动问题，进行了系统地分析和求解，获

得了精确的解析解，他们的工作为开展复合材料层合板壳非线性理论问题的继续研究奠定了坚实的理论基础.

刘人怀等[212-214]研究了更复杂的尚未有人处理的问题，即考虑横向剪切的对称层合圆柱正交异性复合材料截头扁锥壳和线布载荷作用下的扁球壳的非线性稳定问题，用修正迭代法获得了十分精确的解析解. 同时，还对复合材料浅球壳进行了优化设计. 徐加初等[215]进一步研究了三角脉冲作用下对称铺设正交异性复合材料层合扁锥壳的轴对称冲击屈曲，深入讨论了壳体几何参数、物理参数和边界条件对复合材料层合扁锥壳冲击屈曲的影响，并首次将 B-R 准则应用到集中冲击载荷和爆炸冲击载荷作用下的层合扁球壳和层合扁锥壳的非线性动力屈曲问题的研究中[216-217]. 他们指出，当冲击载荷幅值达到某一值时，壳体的位移明显急剧增大，显示壳体发生了屈曲，此时对应的载荷即为临界屈曲载荷. 他们的研究结果表明应用 B-R 准则确定壳体冲击动力屈曲临界载荷是非常有效的. 王璠[218]考虑横向剪切、阻尼、强迫载荷对幅频特性曲线的影响，研究了对称层合圆柱正交异性开孔扁球壳的非线性动态响应. 研究表明，振幅随着阻尼的增大而减小，随着强迫载荷和剪切刚度的增大而增大. 王志伟等[219-221]应用试验和有限元模拟方法分析了蜂窝纸板的动态冲击压缩过程，研究了层数、湿度和厚跨比对蜂窝纸板的冲击承载和能量吸收能力的影响，并首次开展了多次低强度冲击对蜂窝纸板缓冲性能影响的分析. 研究结果表明，随着低强度冲击次数的增加，蜂窝纸板吸能呈上升趋势，蜂窝纸板剩余结构仍能承受冲击作用吸收能量. 宋振华等[222]对机织碳/环氧复合材料在动态压缩载荷作用下的力学行为和失效模式进行了试验分析. Fan 等[223-224]提出了整体夹芯连续纤维复合材料结构动态压缩失效模式和机理.

复合材料不仅给设计者提供高比弹性模量和高比强度的材料，还可以让设计人员在一定范围内对这些性能参数进行改变，以使结构设计和材料使用最优化. 何陵辉等[225]对含非均匀界面相碳/碳纤维复合材料的微结构特性和相应的物理模型进行了详细描述，导出了其等效热传导性质的一些基本关系，得到了等效热传导系数的解析公式，解决了复合材料一个经典难题. 同时，又建立了球面各向同性球粒复合材料的基体均匀场，给出了复合材料等效膨胀系数与体积模量的精确公式[226]. 而且，还建立了空心球增强复合材料的基体均匀场，得到了复合材料局部热弹性场之间以及等效热弹性性质之间的一些精确关系[227]. 王璠等[228]通过制备不同铺层角度和不同引发模式的玻璃纤维/聚酯树脂基圆柱壳，开展了铺层对纤维增强复合材料圆柱壳吸能特性影响的冲击试验研究. 郑伟玲等[229-230]分别介绍了碳纳米管增强复合材料中碳纳米管的制备方法和石墨烯与多壁碳纳米管增强环氧树脂复合材料的制备方法，分析了石墨烯与多壁碳纳米管的协同作用、两者的含量以及多壁碳纳米管功能化方法对复合材料力学和热学性能的影响. 通过将改性碳纳米管按不同质量分数分别加入聚氨酯中，他们研究了碳纳米管/聚氨酯复合材料的制备，讨论了碳纳米管对复合材料性能的影响[231]. 姜久红等[232-233]对蜂窝层合纸板的力学性能进行了研究. 通过静态压缩试验，他们获得了蜂窝层合纸板平压性能，以及芯纸的克重与平压力学性能的关系，讨论了非线性系统中的阻尼比和阻尼系数对产品加速度及位移响应的影响；针对蜂窝纸板在中低应变率下的缓冲性能，他们的研究表明中应变率下蜂窝纸板平台应力和吸能特性随着应变率增大而提高. 应变率对蜂窝纸板的力学性能具有重要影响.

2.2 复合材料板壳结构在复杂环境下的破坏机理

当复合材料处于复杂环境下时，如高温、高压、磁电、湿热、氧化腐蚀和应力耦合，这些环境因素通过不同的机制作用于复合材料，使复合材料的树脂基体、增强纤维以及树脂/纤维粘接界面破坏而引起性能下降, 状态改变, 直至损坏变质.

在湿度和温度的协同作用下，复合材料会产生湿热老化现象，引起其形态、质量、力学性能等指标发生改变. 宁志华等[234-235]首次将复变函数的分区全纯函数理论、Cauchy 型积分与 Riemann 边值问题结合，研究了含非均匀界面层纤维增强复合材料受径向约束的热弹性问题. 基于广义平面应变的假设，考虑材料参数呈线性化变化的非均匀界面层，利用迭代法求解了均匀温变及径向约束下界面层、纤维及基体的位移及应力场. 他们揭示了径向应力约束下各相的位移及应力分布规律，以及界面层的不同体积分数对位移及应力分布的影响，解决了线性温变下含界面层圆形夹杂的温度场难题，所发展的分析方法为求解复杂多连通域的平面热弹性问题提供了一条全新的途径. 除了非均匀界面层纤维增强复合材料的热弹性分析，刘述伦等[236-237]又开展了

正交各向异性复合材料层合板的力学性能受温度和湿度影响的研究，这是一个十分困难的理论问题，而且又有实际意义价值，他们考虑湿－热－力的多场效应，运用宏微观力学模型，分析了考虑含损伤层合板在湿热环境下的分层屈曲问题. 他们的研究展示了当前复合材料非线性问题研究的新特点——多场、多尺度、跨学科，指出了该领域前沿问题的研究新趋势. 同时，朱永安等 [238-239] 运用修正迭代法首次研究了计及横向剪切的对称层合扁球壳的非线性热屈曲问题. E 等 [240-241] 对不同湿度下蜂窝纸板的缓冲性能进行了评估. 他们发现当相对湿度为 40%～75% 时，对蜂窝纸板的缓冲性能无明显影响；当相对湿度超过 75% 时，蜂窝纸板的缓冲性能和平台应力快速下降.

复合材料板壳结构的压电、磁电性能也是当前受到广泛关注的问题之一. 陈炎等 [242-245] 率先研究了压电矩形薄板的非线性自由振动、强迫振动和动态屈曲问题，又建立了位移场和电场高阶分布的压电板的非线性理论. Wang 等 [246-247] 从压电线性理论出发，研究了两种结构在共振频率附近的非线性动力学特征以及在共振频率附近的电–力–电多场非线性耦合作用. 傅衣铭等 [248-249] 研究了具界面损伤压电智能层合板的非线性自由振动. 文献 [250-254] 对受磁–电–机械载荷作用的复合材料层合结构进行了研究.

除了湿热环境和电磁场的影响，高压腐蚀的环境会导致复合材料性能的急剧下降，引起复合材料的结构失稳. 薛江红等 [255] 研究了深海保温夹层输油管道的结构失稳现象，以及腐蚀对管道结构承载能力的影响 [256-258].

2.3 复合材料板壳结构的物理非线性特性

一般而言, 在工程问题中有两种非线性——几何的和物理的. 由于新材料的应用及结构向轻型化发展, 先进复合材料板壳结构往往同时具有几何非线性和材料非线性特性.

纤维增强复合材料已被广泛应用于混凝土结构的加固中，此时复合材料和混凝土材料通过粘结构成层合结构，界面性能对结构破坏往往起到决定性作用 [259-263]. 然而，现有的工作主要涉及最终载荷和有效黏结长度的预测，极少注意到整个剥离过程的研究，因而在很大程度上限制了对界面剥离传播规律和破坏机理的认识. 袁鸿等 [264-267] 基于刚性软化、双线性和三线性的内聚力模型，研究了 FRP 和混凝土界面的剥离破坏. 他们采用线弹性断裂力学和非线性断裂力学理论，提出了一种简便有效的界面能量释放率的计算方法，首次给出了能够预言脱层传播全过程的解析解，建立了与 4 种非线性界面局部剪应力位移关系对应的应力、有效黏结长度、载荷位移关系的解析表达式，阐述了复合材料承载力、载荷位移关系和界面剥离传播之间的关系，提出了界面断裂力学破坏准则. 在此基础上，他们又进一步开展了单向/双向黏贴纤维复合材料抵抗剥落的直梁/曲梁/平板试件的理论和实验研究 [268-270]，分析了不同粘结长度对载荷位移曲线、极限载荷的影响，首次从理论上搞清楚了两条相邻裂纹之间的纤维复合材料补强的混凝土结构的界面性质. 他们的研究为分析全过程载荷位移行为打下了严格和完备的理论基础，对于深入理解复合材料增强结构开裂后的界面粘结层破坏，更好地理解黏结接头的力学特性进而发展有效的复合材料锚固体系十分有益.

由于高温工作环境以及基体材料具有黏性性质，材料力学性能表现出与时间相依的特征. 因此，在对由纤维增强复合材料构成的结构进行动力分析时，需要考虑材料的蠕变特征. 向红等 [271-272] 率先分析了垂直与倾斜壁厚不等的非正六边形不规则蜂窝材料在面内单向应力作用下的蠕变屈曲行为. 王颖坚等 [273] 研究了层合圆柱曲板的蠕变失稳, 得到了确定蠕变失稳临界载荷的解析方法.

2.4 含脱层纤维增强复合材料板壳结构的破坏机理

脱层是复合材料层合板壳结构的一种常见的缺陷或损伤形式. 根据不完全统计，应用于航空航天器的复合材料结构发生破坏时，有接近 60% 的直接原因是脱层. 所以，探究脱层产生的机理、含脱层板壳结构的剩余承载能力，对当前复合材料的科学应用有非凡的意义.

对复合材料板壳结构分层失效破坏机理的研究，是当前非线性力学领域的一个研究热点，受到研究人员的广泛关注. 薛江红等 [274] 将脱层板分成 4 个区域，在冯·卡门非线性理论基础上，建立了复合材料脱层板的屈曲控制方程. 通过各子板连接处位移和力的连续性建立了连续条件. 求得满足边界条件和连续性条件的控制方程的解，并讨论了脱层长度和脱层深度等因素对临界屈曲载荷和屈曲模态的影响. 由于含脱层的纤维增强复合材料板承受面内和面外载荷作用时，会在脱层界面上产生接触现象，引起接触力. 如果忽略接触效应可能导致层合

子板相互之间的互相贯穿. 因此，薛江红等 [275] 又开展了含脱层层合板在面外载荷作用下的接触问题的研究. 通过分析脱层界面处的变形机制，运用复合材料微观力学，他们建立了脱层界面处上下子板变形挠度与接触力之间的关系，并采用摄动法求解了受面外载荷作用的含脱层层合板的大挠度弯曲问题，首次获得了接触效应的解析解，解决了脱层板子板之间接触效应，包括接触力的大小、接触区域的大小和位置等难于确定的难题. 他们首次发现，位于分层界面的上下两个子板的挠度由两部分组成：完全一致的整体弯曲挠度和相反的局部微小屈曲模态；接触区域的位置和大小，由脱层所在的位置和大小决定，而与层合板的长宽比无关. 据此，他们进一步研究了考虑接触影响的含脱层层合板受面内载荷作用的屈曲问题 [276]，并与不考虑接触影响的情况进行对比 [274]. 他们的研究结果表明，在分析含脱层的复合材料层合板的屈曲问题时，是否考虑接触效应会对结果产生显著的区别. 考虑接触效应的层合板的分层屈曲压力远高于不考虑接触效应的层合板的分层屈曲压力. 最近，刘述伦等 [237] 又研究了含脱层复合材料层合矩形板在湿热环境下的屈曲问题. 此外，Zhu 等 [277] 提出了一种压电层合圆柱壳的非线性接触效应分析模型，考虑了非线性接触效应对能量释放率和分层增长率的影响，同时他们 [278-281] 研究脱层的层合圆板的非线性动力响应、脱层层合圆柱壳的屈曲和振动等问题. Chen 等 [282-283] 用一阶剪切变形理论，Schwarts-Givli 等 [284] 和 Oh 等 [174] 用高阶剪切变形理论，Wang 等 [285] 用非线性反贯穿模型，以及 Kwon 等 [286] 用考虑接触冲击条件的有限元模拟等方法分别研究了接触对分层复合材料结构的动力性能的影响. 文献 [287-293] 给出了更多关于含脱层复合材料层合板壳的非线性力学研究成果.

3 研究展望

在详细介绍过去复合材料层合板壳非线性力学研究进展的基础上，本文对今后的研究方向提出了如下的展望.

3.1 等效单层板壳理论与多层板壳理论相结合的理论分析法

目前, 对复合材料层合板壳的分析理论有两个方向：等效的单层板壳理论和多层板壳理论. 等效的单层板壳理论将层合板壳视为一个具有复杂的本构关系的单层板壳，从最基本的 Kirchhoff 理论开始，在位移场上叠加沿厚度的高阶展开项. 虽然位移场的阶数越高，得到的解的精度也越高，但各高阶项所对应的力学属性却难以确定. 事实上，目前尚不清楚三阶以上各项的力学描述. 而大量的研究结果表明，用等效单层板壳理论得到的层合板壳的整体性能分析与用三维连续介质理论得到的结果及实验结果有极高的吻合性. 多层板壳理论则将层合板壳化为多个子层，对每个子层独立进行研究分析. 研究表明，多层板壳理论能准确描述层合板壳的局部力学性能，但子层间位移和应力连续的必要条件导致随着子层数的增加计算量大大增加.

在层合板壳理论未来的研究发展中，上述两个方向将共存，而一个具体方法的使用取决于微观或宏观层面的研究，甚而两个方向的结合.

3.2 含损伤复合材料板壳结构在复杂环境下的失效分析

对含损伤复合材料层合板壳结构在外载作用下的失效机理研究将导致多学科之间的交叉渗透. 如分层复合材料在湿、热、机械力共同作用下的板壳结构整体稳定性问题，需要同时考虑弹性稳定性问题、接触问题和断裂问题. 而湿热环境对复合材料层合板壳结构机械性能的影响又具有多场耦合的效果. 深入分析在复杂环境中不完善层合板壳结构的失效机理，具有多尺度、多场耦合、跨学科的特点，不仅可在复合材料层合板壳结构力学的分析理论上取得突破进展，而且对复合材料在实际工程中的应用具有重要的指导意义.

3.3 不完善复合材料层合板壳结构的可靠性与优化设计

复合材料具有良好的材料性能可设计性、制备的灵活性和易加工性. 设计者可以通过改变组成材料的种类、含量和铺设方向，来达到在结构设计中对材料刚度、强度、稳定性和方向性的要求. 但复合材料的力学指标受组分材料、界面性质、层合结构、载荷、环境等多种因素的影响, 并含有许多不确定因素，而且含缺陷的不完善复合材料的失效具有多种模式，如基体开裂、脱胶、分层、分层裂纹偏转、多分层以及分层传播等等，当存在多种失效模式时, 需要建立针对各种破坏机理的失效模型，确定其主要失效模式以及建立相应的可靠性指标对其进行评估. 由于问题的复杂性，目前的研究报道有限，尚需进一步研究.

参 考 文 献

1 刘人怀. 板壳力学. 北京：机械工业出版社, 1990 (Liu Renhuai. Mechanics of Plates and Shells. Beijing: Machinery Industry Press, 1990 (in Chinese))

2 徐芝纶. 弹性力学 (下). 北京：高等教育出版社, 2006 (Xu Zhilun. Elasticity. Beijing: Higher Education Press, 2006 (in Chinese))

3 沈观林，胡更开. 复合材料力学. 北京：清华大学出版社, 2006 (Shen Guanlin, Hu Gengkai. Mechanics of Composite Materials. Beijing: Tsinghua University Press, 2006 (in Chinese))

4 von Kármán Th.V. Festigkeits Problem in Maschinenban, Enzyklopädie der Mathematjschen Wissenschaften, Bd. IV., art. 27,1910

5 Liu RH. Study on Nonlinear Mechanics of Plates and Shells. New York, Beijing, Guangzhou: Science Press and Jinan University Press, 1998

6 刘人怀. 复合材料层合板壳理论探索. 广州: 暨南大学出版社, 2006 (Liu Renhuai. Probe into the Theory of Laminated Composite Plates and Shells. Guangzhou: Jinan University Press, 2006 (in Chinese))

7 Stavsky Y. On the general theory of heterogeneous aeolotropic plates. *Aeronautic Quart*, 1964, 15: 29

8 Whitney JM, Leissa AW. Analysis of heterogenous anisotropic plates. *ASME Journal of Applied Mechanics*, 1968, 36(3): 261-266

9 Chia CY. Nonlinear Analysis of Plates. New York: McGraw-Hill, 1980

10 Chia CY. Large deflection of heterogeneous anisotropic rectangular plates. *International Journal of Solids and Structures*, 1974, 10(9): 965-976

11 Chia CY, Prabhakara MK. Large deflection of unsymmetric cross-ply and angle-ply plates. *ARCHIVE Journal of Mechanical Engineering Science*, 1976, 18(3): 179-183

12 Prabhakara MK, Chia CY. Finite deflections of unsymmetrically layered anisotropic rectangular plates subjected to the combined action of transverse and inplane loads. *ASME Journal of Applied Mechanics*, 1975, 42(2): 517-518

13 Prabhakara MK, Chia CY. Nonlinear analysis of laminated cross-ply plates. *Journal of the Engineering Mechanics Division*, 1977, 103(3): 749-753

14 Prabhakara MK, Chia CY. Non-linear flexural vibrations of orthotropic rectangular plates. *Journal of Sound and Vibration*, 1977, 52(3): 511-518

15 Prabhakara MK. Finite deflection of unsymmetric angle-ply anisotropic rectangular plates under edge moments. *ASME Journal of Applied Mechanics*, 1977, 44(1): 171-172

16 Chia CY. Nonlinear bending of unsymmetric angleply plates with edges elastically restrained against rotation. *Acta Mechanica*, 1984, 53(3-4): 201-212

17 Chia CY. Large deflection of unsymmetric laminates with mixed boundary conditions. *International Journal of Non-Linear Mechanics*, 1985, 20(3): 273-282

18 叶开沅, 刘人怀, 平庆元等. 圆底扁薄球壳的非线性稳定问题 —— 在对称线布载荷作用下的圆底扁薄球壳的非线性稳定简题. 科学通报, 1965, 16(2): 142-145 (Ye Kaiyuan, Liu Renhuai, Ping Qingyuan, et al. Nonlinear stability of shallow spherical shells with round bottom-nonlinear stability of thin spherical shells with circular bottom under symmetrical linear load. *Chinese Science Bulletin*. 1965, 16(2): 142-145 (in Chinese))

19 叶开沅, 刘人怀, 张传智等. 圆底扁薄球壳的非线性稳定问题 —— 圆底扁薄球壳在边缘力矩作用下的非线性稳定问题. 科学通报, 1965, 16(2): 145-147 (Ye Kaiyuan, Liu Renhuai, Zhang Chuanzhi, et al. Nonlinear stability of shallow spherical shells with round bottom-nonlinear stability of thin spherical shells with circular bottom under the action of edge moment. *Chinese Science Bulletin*, 1965, 16(2): 145-147 (in Chinese))

20 刘人怀. 在内边缘均布力矩作用下中心开孔圆底扁球壳的非线性稳定问题. 科学通报，1965, 16(3): 253-255 (Liu Renhuai. Nonlinear stability of a shallow spherical shell with a central opening under the action of the uniformly distributed moments at the interior edge. *Chinese Science Bulletin*, 1965, 16(3): 253-255 (in Chinese))

21 刘人怀. 在内边缘均布载荷作用下中心开孔圆底扁球壳的轴对称稳定性. 力学学报，1977, 13(2): 206-212 (Liu Renhuai. Axisymmetric stability of a shallow spherical shell with a central opening under uniformly distributed load at the interior edge. *Acta Mechanica Sinica*, 1977, 13(2): 206-212 (in Chinese))

22 刘人怀. 波纹圆板的特征关系式. 力学学报，1978, 14(1): 47-52 (Liu Renhuai. Characteristic formula of corrugated circular plates. *Acta Mechanica Sinica*, 1978, 14(1): 47-52 (in Chinese))

23 叶开沅, 刘人怀, 张传智等. 弹性圆底扁球壳在边缘均布力矩作用下的非线性稳定问题. 应用数学和力学, 1980, 1(1): 71-87 (Yeh Kaiyuan, Liu Renhuai, Zhang Chuanzhi, et al. Nonlinear stabi1ity of thin elastic circular shallow spherical shell under the action of uniform edge moment. *Applied Mathematics and Mechanics*, 1980, 1(1): 71-87 (in Chinese))

24 刘人怀. 夹层圆板的非线性弯曲. 应用数学和力学，1981, 2(2): 173-190 (Liu Renhuai. Nonlinear bending of circular sandwich plates. *Applied Mathematics and Mechanics*, 1981, 2(2): 173-190 (in Chinese))

25 Liu RH. Nonlinear thermal stability of bimetallic shallow shells of revolution. *International Journal of Non-Linear Mechanics*, 1983, 18(4): 409-429

26 Liu RH. Large deflection of corrugated circular plate with a plane central region under the action of concentrated loads at the center. *International Journal of Non-Linear Mechanics*, 1984, 19(4): 409-415

27 刘人怀. 波纹环形板的非线性弯曲. 中国科学 (A 辑), 1984, (2): 247-253 (Liu Renhuai. Nonlinear bending of corrugated annular plates. *Science in China Series A*, 1984, (2): 247-253 (in Chinese))

28 Liu RH. Large deflection of corrugated circular plate with plane boundary region. *Solid Mechanics Archives*, 1984, 9(3): 383-406

29 刘人怀. 在复杂载荷作用下波纹环形板的非线性弯曲. 中国科学 (A 辑), 1985 (6): 537-545 (Liu Renhuai. Nonlinear analysis of corrugated annular plates under compound load. *Science in China Series A*, 1985 (6): 537-545 (in Chinese))

30 Liu RH. Nonlinear bending of circular sandwich plates under the action of axisymmetric uniformly distributed line loads. *Progress in Applied Mechanics*, Dordrecht: Martinns Nijhoff Publishers, 1987, 293-321

31 刘人怀, 成振强. 集中载荷作用下开顶扁球壳的非线性稳定问题. 应用数学和力学，1986, 9(2): 95-105 (Liu Renhuai, Cheng Zhen-

qiang. On the nonlinear stability of truncated shallow spherical under a concentrated load. *Applied Mathematics and Mechanics*, 1986, 9(2): 95-105 (in Chinese))

32 刘人怀, 李东. 均布载荷作用下开顶扁球壳的非线性稳定问题. 应用数学和力学, 1988, 9(2): 205-217 (Liu Renhuai, Li Dong. On the nonlinear stability of truncated shallow spherical under a uniformly load. *Applied Mathematics and Mechanics*, 1988, 9(2): 205-217 (in Chinese))

33 刘人怀. 复合载荷作用下波纹圆板的非线性分析. 应用数学和力学，1988, 9(6): 661-674 (Liu Renhuai. Nonlinear analysis of a corrugated circular plate under combined lateral loading. *Applied Mathematics and Mechanics*, 1988, 9(6): 661-674 (in Chinese))

34 Liu RH, He LH. On the nonlinear stability of a truncated shallow spherical shell under axisymmetrically distributed load. *Solid Mechanics Archives*, 1989, 14(2): 81-102

35 Liu RH, Li D. On the nonlinear bending and vibration of corrugated circular plates. *International Journal Non-linear Mechanics*, 1989, 24(2): 165-176

36 刘人怀, 朱金福. 夹层壳非线性理论. 北京：机械工业出版社，1993 (Liu Renhuai, Zhu Jinfu. Nonlinear Theory of Sandwich Shell. Beijing: Machinery Industry Press, 1993 (in Chinese))

37 刘人怀. 精密仪器仪表弹性元件的设计原理. 广州：暨南大学出版社，2006 (Liu Renhuai. Principle for Designing Elastic Elements in Precision-Lnstruments Manufacture. Guangzhou: Jinan University Press, 2006 (in Chinese))

38 刘人怀. 夹层板壳非线性理论分析. 广州：暨南大学出版社，2007 (Liu Renhuai. Nonlinear Theory and Analysis of Sandwich Plates and Shells. Guangzhou: Jinan University Press, 2007 (in Chinese))

39 刘人怀. 网壳结构的非线性弯曲、稳定和振动. 北京：科学出版社，2011 (Liu Renhuai. Nonlinear bending, stability and vibration of reticulated shells. Beijing: Science Press, 2011 (in Chinese))

40 刘人怀，何陵辉. 四边简支对称正交层合矩形板的非线性弯曲问题. 应用数学和力学，1990, 11(7): 753-759 (Liu Renhuai, He Linghui. Nonlinear bending of simply supported symmetric laminated cross-ply rectangular plates. *Applied Mathematics and Mechanics*, 1990, 11(7): 753-759 (in Chinese))

41 刘人怀. 对称圆柱正交异性层合扁球壳的非线性稳定问题. 应用数学和力学，1991, 12(2): 252-258 (Liu Renhuai. On the nonlinear stability of symmetrically laminated cylindrically orthotropic shallow spherical shells. *Applied Mathematics and Mechanics*, 1991, 12(2): 252-258 (in Chinese))

42 刘人怀. 复合材料层合扁锥壳的非线性稳定问题. 第九届全国复合材料学术会议论文集. 北京：世界图书出版社公司，1996: 249-255 (Liu Renhuai. Nonlinear stability of composite laminated shallow conical shells//Proceedings of the Ninth National Conference on Composite Materials. Beijing: World Books Publishing Company, 1996: 249-255 (in Chinese))

43 Turvey GJ, Wittrick WH. The large deflection and postbuckling behavior of some laminated plates. *Aeronaut Q*, 1973, 24: 77

44 Zaghloul SA, Kennedy JB. Nonlinear behavior of symmetrically laminated plates. *Journal of Applied Mechanics*, 1975, 42(1): 234-236

45 Chandra R. Nonlinear bending of antisymmetric angle-ply laminated plates. *Fibre Science & Technology*, 1977, 10(2): 123-137

46 周次青. 对称正交铺设矩形叠层板的非线性弯曲. 应用数学和力学, 1985, 6(7): 819-832 (Zhou Ciqing. Nonlinear bendings of rectangular symmetrically laminated cross-ply plates under various supports. *Applied Mathematics and Mechanics*, 1985, 6(7): 819-832 (in Chinese))

47 周次青. 不对称的各向异性叠层矩形板的非线性弯曲. 应用数学和力学, 1986, 7(11): 1003-1020 (Zhou Ciqing. Nonlinear bendings of unsymmetrically layered anisotropic rectangular plates. *Applied Mathematics and Mechanics*, 1986, 7(11): 1003-1020 (in Chinese))

48 周次青. 对称铺设各向异性叠层矩形板的非线性弯曲. 应用数学和力学, 1988, 9(2): 267-280 (Zhou Ciqing. Nonlinear bendings of symmetrically layered anisotropic rectangular plates. *Applied Mathematics and Mechanics*, 1988, 9(2): 267-280 (in Chinese))

49 Reissner E. The effect of transverse shear deformation on the bending of elastic plates. *Journal of Applied Mechanics*, 1945, 12: 69-77

50 Mindlin RD. Influence of rotatory inertial and shear on flexural motion of isotropic elastic plates. *Journal of Applied Mechanics*, 1951, 18(1): 31-38

51 Stavsky Y. On the theory of heterogeneous anisotrophic plates. [PhD Thesis]. MIT, Cambridge, MA, 1959

52 Yang P, Norris CH, Stavsky Y. Elastic wave propagation in heterogeneous plates. *International Journal of Solids and Structures*, 1966, 2(4): 665-684

53 Ambartsumyan SA. Theory of Anisotropic Plates. Stamford: Technomic Publishing Co., Inc, 1970

54 Bert CW. Nonlinear vibration of a rectangular plate arbitrarily laminated of anisotropic materials. *Journal of Applied Mechanics*, 1973, 40(2): 452-458

55 Whitney JM. Structural analysis of laminated anisotropic plates. Stamford: Technomic Publishing Co., Inc, 1987

56 Dong SB, Tso FKW. On a laminated orthotropic shell theory including transverse shear deformation. *Journal of Applied Mechanics*, 1972, 39(4): 1091-1097

57 Reddy JN. A small strain and moderate rotation theory of laminated anisotropic plates. *Journal of Applied Mechanics*, 1987, 54(3): 623-626

58 Schmidt R, Reddy JN. A refined small strain and moderate rotation theory of elastic anisotropic shells. *Journal of Applied Mechanics*, 1988, 55(3): 611-617

59 Palmerio AF, Reddy JN, Schmidt R. On a moderate rotation theory of elastic anisotropic shells. Part 1. Theory. *International Journal of Non-Linear Mechanics*, 1990, 25(6): 687-700

60 Palmerio AF, Reddy JN, Schmidt R. On a moderate rotation theory of elastic anisotropic shells. Part 2. FE analysis. *International Journal of Non-Linear Mechanics*, 1990, 25(6): 701-714

61 Kreja I, Schmidt R, Reddy JN. Finite elements based on a first-order shear deformation moderate rotation shell theory with applications to the analysis of composite structures. *International Journal of Non-Linear Mechanics*, 1991, 32(6): 1123-1142

62 Iu VP, Chia CY. Effect of transverse shear on nonlinear vibration and postbuckling of anti-symmetric cross-ply imperfect cylindrical shells. *International Journal of Mechanical Sciences*, 1988, 30(8): 705-718

63 Fu YM, Chia CY. Non-linear vibration and postbuckling of generally laminated circular cylindrical thick shells with non-uniform boundary conditions. *International Journal of Non-Linear Mechanics*, 1993, 28(3): 313-327

64 Fu YM, Chia CY. Multi-mode nonlinear vibration and postbuckling of anti-symmetric imperfect angle-ply cylindrical thick panels. *International Journal of Non-Linear Mechanics*, 1989, 24(5): 365-381

65 Sivakumaran KS, Chia CY. Large-amplitude oscillations of unsymmetrically laminated anisotropic rectangular plates including shear, rotatory inertia and transverse normal strains. *ASME Journal of Applied Mechanics*, 1985, 52(3): 536-542

66 Xu CS, Xia ZQ, Chia CY. Non-linear theory and vibration analysis of laminated truncated, thick, conical shells. *International Journal of Non-Linear Mechanics*, 1996, 31(2): 139-151

67 Fares ME. Non-linear bending analysis of composite laminated plates using a refined first-order theory. *Composite Structures*, 1999, 46(3): 257-266

68 Fares ME, Zenkour AM. Mixed variational formula for the thermal bending of laminated plates. *Journal of Thermal Stresses*, 1999, 22(2): 347-365

69 Fares ME. Mixed variational formulation in geometrically nonlinear elasticity and a generalized nth-order beam theory. *International Journal of Non-Linear Mechanics*, 1999, 34(2): 685-691

70 Fares ME. Generalized non-linear thermoelasticity for composite laminated structures using a mixed variational approach. *International Journal of Non-Linear Mechanics*, 2000, 35(2): 439-446

71 Whiteny JM, Pagano NJ. Shear deformation in heterogeneous anisotropic plates. *Journal of Applied Mechanics*, 1970, 37(4): 1031-1036

72 Fortier RC, Rossettos JN. On the vibration of shear deformable curved anisotropic composite plates. *Journal of Applied Mechanics*, 1973, 40(1): 299-301

73 Sinha PK, Rath AK. Vibration and buckling of cross-ply laminated circular cylindrical panels. *Aeronaut Quart*, 1975, 26: 211-218

74 Bert CW, Chen TL. Effect of shear deformation on vibration of antisymmetric angle-ply laminated rectangular plates. *International Journal of Solids and Structures*, 1978, 14(6): 465-473

75 刘人怀. 考虑横向剪切的对称层合圆柱正交异性扁球壳的非线性稳定问题. 中国科学 (A 辑), 1991 (7): 742-751 (Liu Renhuai. Nonlinear stability of symmetric laminated orthotropic cylindrical shallow spherical shells with transverse shear. *Science in China, Series. A*, 1991 (7): 742-751 (in Chinese))

76 刘人怀，王璠. 复合材料层合扁球壳的非线性强迫振动. 力学学报，1997, 29(2): 236-241 (Liu Renhuai, Wang Fan. Nonlinear forced vibration of laminated composite shallow spherical shells. *Acta Mechanica Sinica*, 1997, 29(2): 236-241 (in Chinese))

77 Liu RH, Wang F. Nonlinear dynamic buckling of symmetrically laminated cylindrically orthotropic shallow spherical shells. *Archive of Applied Mechanics*, 1998, 68(6): 375-384

78 刘人怀, 徐加初. 直线型正交异性层合圆板的大挠度问题. 暨南大学学报, 1998, 19(4): 1-10 (Liu Renhuai, Xu Jiachu. Large deflection of symmetrically laminated rectilinearly orthotropic circular plates. *Journal of Jinan University*, 1998, 19(4): 1-10 (in Chinese))

79 刘人怀，徐加初. 剪切变形对直线型正交异性层合圆板大幅度受迫振动的影响. 应用数学和力学，1998, 19(2): 105-112 (Liu Renhuai, Xu Jiachu. Shear effects on large amplitude forced vibration of symmetrically laminated rectilinearly orthotropic circular plates. *Applied Mathematics and Mechanics*, 1998, 19(2): 105-112 (in Chinese))

80 Liu RH, Xu JC, Zhai SZ. Large-deflection bending of symmetrically laminated rectilinearly orthotropic elliptical plates including transverse shear. *Archive of Applied Mechanics*, 1997, 67(7): 507-520

81 Liu RH. On non-linear buckling of symmetrically laminated, cylindrical orthotropic, truncated, shallow, spherical shells considering uniform pressure including shear effects. *International Journal of Non-Linear Mechanics*, 1996, 31(1): 101-115

82 刘人怀，钟诚. 考虑横向剪切变形的对称层合圆柱正交异性中心开孔扁球壳的非线性屈曲. 暨南大学学报, 1994, 15(1): 1-11 (Liu Renhuai, Zhong Cheng. Nonlinear buckling of symmetrically laminated cylindrically orthotropic shallow spherical shells with a circular hole at the center including transverse shear. *Journal of Jinan University*, 1994, 15(1): 1-11 (in Chinese))

83 刘人怀，王璠，肖谭. 阻尼对层合复合材料中心开孔扁球壳非线性振动的影响. 刘人怀等主编. 应用力学研究与实践, 广州：暨南大学出版社，2000: 35-43 (Liu Ren-Huai, Wang Fan, Xiao Tan. Effect of damping on nonlinear vibration of Laminated Shallow Spherical Shell with center opening//Liu Renhuai et al. eds. Research and Practice in Applied Mechanics, Guangzhou: Jinan University Press, 2000: 35-43 (in Chinese))

84 王璠, 刘人怀. 复合材料层合开顶扁球壳的非线性动态屈曲，固体力学学报, 2001, 22(2): 309-314 (Wang Fan, Liu Renhuai. Nonlinear dynamic buckling of symmetrically laminated truncated spherical shell. *Acta Mechanica Solida Sinica*, 2001, 22(2): 309-314 (in Chinese))

85 Liu RH. Non-linear buckling of symmetrically laminated, cylindrically orthotropic, shallow, conical shells considering shear. *International Journal of Non-Linear Mechanics*, 1996, 31(1): 89-99

86 刘人怀，王志伟. 复合材料面层夹层板中转动一致有效理论. 上海力学，1996, 17(2): 222-228 (Liu Renhuai, Wang Zhiwei. The theory of consistent moderate rotation of sandwich plates faced with laminated composites. *Shanghai Journal of Mechanics*, 1996, 17(2): 222-228 (in Chinese))

87 刘人怀，王志伟. 考虑横向剪切应力连续的复合材料面层夹层壳非线性一致有效理论. 刘人怀等主编. 应用力学研究与实践, 广州：暨南大学出版社, 2000: 1-12 (Liu Renhuai, Wang Zhiwei. Nonlinear consistent theory for sandwich plates of laminated composite faces with transverse shear stress// Liu Renhuai et al. eds. Research and Practice in Applied Mechanics, Guangzhou: Jinan University Press, 2000: 1-12 (in Chinese))

88 王志伟，刘人怀. 正交复合材料面层夹层板非线性理论及应用. 应用力学学报，1998, 15(2): 18-24 (Wang Zhiwei, Liu Renhuai. A refined nonlinear theory for sandwich plates faced with orthotropic laminated composites and its application. *Chinese Journal of Applied Mechanics*, 1998, 15(2): 18-24 (in Chinese))

89 Wang ZW, Liu RH. Nonlinear buckling for sandwich shallow shells accounting for continuity conditions of stresses at interfaces//Proceedings of the 3rd Int. Conf. on Nonlinear Mechcanics, Shanghai: Shanghai University Press, 1988: 381-386

90 王志伟，刘人怀. 复合材料面层夹层扁壳非线性精化理论及应用. 应用力学学报，2000, 17(3): 86-91 (Wang Zhiwei, Liu Renhuai. A new refined nonlinear theory for sandwich shallow with orthotropic laminated faces and its application. *Chinese Journal of Applied Mechanics*, 2000, 17(3): 86-91 (in Chinese))

91 徐加初，王乘，刘人怀. 具有夹层的正交异性复合材料圆锥壳体的非线性轴对称屈曲分析. 工程力学, 2001, 18(3): 28-37 (Xu Jiachu, Wang Cheng, Liu Renhuai. Nonlinear axisymmetric buckling of conical sandwich shells with orthotropic skins. *Engineering Mechanics*, 2001, 18(3): 28-37 (in Chinese))

92 Chow TS. On the propagation of flexural waves in an orthotropic laminated plate and its response to an impulsive load. *Journal of Composite Materials*, 1971, 5(3): 306-319

93 Whitney JM. Shear correction factors for orthotropic laminates under static load. *Journal of Applied Mechanics*, 1973, 40(1): 302-304

94 Noor AK, Burton WS. Assessment of shear deformation theories for multilayered composite plates. *Applied Mechanics Review*, 1989, 42(1): 1-13

95 Noor AK, Burton WS. Stress and free vibration analyses of multilayered composite plates. *Composite Structures*, 1989, 11(3): 183-204

96 Vlachoutsis S. Shear correction factors for plates and shells. *International Journal for Numerical Methods in Engineering*, 1992, 33(7): 1537-1552

97 Laitinen M, Lahtinen H, Sjolind SG. Transverse shear correction factors for laminates in cylindrical bending. *Communications in Numerical Methods in Engineering*, 1995, 11(1): 41-47

98 Reddy JN. A simple higher-order theory for laminated composites. *Journal of Applied Mechanics*, 1984, 51(3): 745-752

99 Reddy JN. Energy and Variational Methods in Applied Mechanics with an Introduction to the Finite-Element Method. New York: Wiley, 1984: 364-371

100 Reddy JN. A refined nonlinear theory of plates with transverse shear deformation. *International Journal of Solids and Structures*, 1984, 20(9-10): 881-896

101 Reddy JN, Liu CF. A higher-order shear deformation theory of laminated elastic shells. *International Journal of Engineering Science*, 1985, 23(3): 319-330

102 Simitses GJ, Anastasiadis JS. Shear deformable theories for cylindrical laminates — equilibrium and buckling with applications. *AIAA Journal*, 1992, 30(3): 826-834

103 Huang NN. Influence of shear correction factors in the higher order shear deformation laminated shell theory. *International Journal of Solid and Structures*, 1994, 31(9): 1263-1277

104 Sheikh AH, Chakrabarti A. A new plate bending element based on higher-order shear deformation theory for the analysis of composite plates. *Finite Elements in Analysis and Design*, 2003, 39(7): 883-903

105 Nayak AK, Moy SSJ, Shenoi RA. Free vibration analysis of composite sandwich plates based on Reddy's higher-order theory. *Composites: Part B*, 2002, 33(7): 505-519

106 Nayak AK, Moy SSJ, Shenoi RA. A higher order finite element theory for buckling and vibration analysis of initially stressed composite sandwich plates. *Journal of Sound and Vibration*, 2005, 286(4-5): 763-780

107 Shahrokh HH, Fadaee M, Taher HRD. Exact solutions for free flexural vibration of Levy-type rectangular thick plates via third-order shear deformation plate theory. *Applied Mathematical Modelling*, 2011, 35(2): 708-727

108 Di S, Rothert H. Solution of laminated cylindrical shells using an unconstrained third-order theory. *Composite Structures*, 1995, 32(4): 667-680

109 Nayak AK, Moy SJ, Shenoi RA. Free vibration analysis of composite sandwich plates based on Reddy's higher-order theory. *Composites: Part B—Engineering*, 2002, 33(7): 505-519

110 Kant T, Swaminathan K. Free vibration of isotropic, orthotropic, and multilayer plates based on higher order refined theories. *Journal of Sound and Vibration*, 2001, 241(2): 319-327

111 Kant T, Swaminathan K. Analytical solutions for free vibration of laminated composite and sandwich plates based on a higher-order refined theory. *Composite Structures*, 2001, 53(1): 73-85

112 Kant T, Swaminathan K. Analytical solutions for the static analysis of laminated composite and sandwich plates based on a higher order refined theory. *Composite Structures*, 2002, 56(4): 329-344

113 Barut A, et al. Analysis of thick sandwich comstruction by a {3,2}-order theory. *International Journal of Solids and Structures*, 2001, 38(34-35): 6063-6077

114 Liu RH, He LH. A simple theory for non-linear bending of laminated composite rectangular plates including higher-order effects. *International Journal of Non-Linear Mechanics*, 1991, 26(4): 537-545

115 Kuppusamy T, Reddy AN. A three-dimensional nonlinear analysis of cross-ply rectangular composite plates. *Computers and Structures*, 1984, 18(2): 263-272

116 沈惠申. 高阶剪切变形板理论 Karman 型方程及在热后屈曲分析中的应用. 应用数学和力学, 1997, 18(9): 1059-1073 (Shen Huishen. Karman-type equations for a higher-order shear deformation plate theory and its use in the thermal postbuckling analysis. *Applied Mathematics and Mechanics*, 1997, 18(9): 1059-1073 (in Chinese))

117 沈惠申. 复合材料层合剪切圆柱曲板在侧压作用下的后屈曲. 应用数学和力学, 2003, 24(3): 357-366 (Shen Huishen. Postbuckling of pressure-loaded shear deformable laminated cylindrical panels. *Applied Mathematics and Mechanics*, 2003, 24(3): 357-366 (in Chinese))

118 冯世宁, 陈浩然. 基于高阶剪切理论的层合板非线性动力稳定性. 大连理工大学学报, 2006, 6(1): 1-6 (Feng Shining, Chen Haoran. Analysis of nonlinear dynamic stability of composite laminates based on higher order shear deformation theory. *Journal of Dalian University of Technology*, 2006, 6(1): 1-6 (in Chinese))

119 张雨, 向锦武. 复合材料层合厚圆柱壳高阶理论的改进及其应用. 复合材料学报, 2002, 19(3): 86-91 (Zhang Yu, Xiang Jinwu. Higher-order theory and its finite element method for thick laminated composite cylindrical shells. *Acta Materiae Compositae Sinica*, 2002, 19(3): 86-91 (in Chinese))

120 Whitney JM, Sun CT. A higher order theory for extensional motion of laminated composites. *Journal of Sound and Vibration*, 1973, 30(1): 85-97

121 傅晓华, 陈浩然, 王振鸣. 复合材料多层厚板的精化高阶理论及其有限元法. 复合材料学报, 1992, 9(2): 39-47 (Fu Xiaohua, Chen Haoran, Wang Zhenming. A refined higher-order theory and its finite element method for thick laminated plates. *Acta Materiae Compositae Sinica*, 1992, 9(2): 39-47 (in Chinese))

122 范业立, 曾加雄. 筒形弯曲和球壳弯曲的一个新的高阶理论解. 华南理工大学学报, 1991, 19(4): 44-53 (Fan Yeli, Zeng Jiaxiong. A new higher-order theory of laminated plates to cylindrical bending and spherical shell bending. *Journal of South China University of Technology*, 1991, 19(4): 44-53 (in Chinese))

123 陈浩然, 温玄玲, 左庄太. 含分层损伤轴对称复合材料层合壳的高阶理论及有限元分析. 大连理工大学学报, 1999, 39(5): 601-606 (Chen Haoran, Wen Xuanling, Zuo Zhuangtai. A high-order theory and finite element analysis for axis-symmetrical composite laminated shell with delamination. *Journal of Dalian University of Technology*, 1999, 39(5): 601-606 (in Chinese))

124 Matsunaga H. Assessment of a global higher-order deformation theory for laminated composite and sandwich plates. *Composite Structures*, 2002, 56(3): 279-291

125 Matsunaga H. A comparison between 2-D single-layer and 3-D layerwise theories for computing interlaminar stresses of laminated composite and sandwich plates subjected to thermal loadings. *Composite Structures*, 2004, 64(2): 161-177

126 Matsunaga H. Interlaminar stress analysis of laminated composite and sandwich circular arches subjected to thermal/mechanical loading. *Composite Structures*, 2003, 60(2): 345-358

127 Matsunaga H. Stress analysis of functionally graded plates subjected to thermal and mechanical loadings. *Composite Structures*, 2009, 87(3): 344-357

128 Matsunaga H. Effects of higher order deformations on in-plane vibration and stability of thick circular rings. *Acta Mechanica*, 1997, 124(1): 47-61

129 Matsunaga H. Vibration and buckling of multilayered composite beam according to higher order deformation theories. *Journal of Sound and Vibration*, 2001, 246(l): 47-56

130 Reissner E. A twelfth-order theory of transverse bending of transversely isotropic plates. *Zeitschrift für Angewandte Mathematic und Mechanik*, 1983, 63(7): 285-289

131 Ambartsumian SA. On theory of bending plates. *Isz Otd Tech Nauk AN SSSR*, 1958, 5: 69-77

132 Kaczkowski Z. Plates-Statical Calculations. Arkady: Warsaw, 1968

133 Panc V. Theories of Elastic Plates. Prague: Academia, 1975

134 Levinson M. An accurate simple theory of the statics and dynamics of elastic plates. *Mechanics Research Communications*, 1980, 7(6): 343-350

135 Ghugal YM, Sayyad AS. Free vibration of thick orthotropic plates using trigonometric shear deformation theory. *Latin American Journal of Solids and Structures*, 2011, 8(3): 229-243

136 El Meiche N, Tounsi A, Ziane N, et al. A new hyperbolic shear deformation theory for buckling and vibration of functionally graded sandwich plate. *International Journal of Mechanical Science*, 2011, 53 (3): 237-247

137 Matsunaga H. Vibration and stability of cross-ply laminated composite plates according to a global higher-order plate theory. *Composite Structures*, 2000, 48(3): 231-244

138 Karama M, Afaq KS, Mistou S. Mechanical behavior of laminated composite beam by the new multilayered laminated composite structures model with transverse shear stress continuity. *International Journal of Solid and Structures*, 2003, 40(6): 1525-1546

139 Karama M, Afaq KS, Mistou S. A new theory for laminated composite Plates//Proceedings of the IMechE Conference 2009. 223

140 Karama M, Abouharb B, Mistou S, et al. Bending, buckling and free vibration of laminated composite with transverse shear stress continuity model. *Composites: Part B-Engineering*, 1998, 29B: 223-234

141 Aydogdu M. Comparison of various shear deformation theories for bending, buckling and vibration of rectangular symmetric cross-ply plate with simply supported edges. *Journal of Composite Materials*, 2006, 40(23): 2143-2155

142 Aydogdu M. A new shear deformation theory for laminated composite plates. *Composite Structures*, 2009, 89(1): 94-101

143 Neves AMA, Ferreira AJM, Carrera E, et al. A quasi-3D sinusoidal shear deformation theory for the static and free vibration analysis of functionally graded plates. *Composites Part B–Engineering*, 2012, 43(2): 711-725

144 Murakami H. Laminated composite plate theory with improved in-plane response. *Journal of Applied Mechanics*, 1986, 53(3): 661-666

145 Murakami H, Toledano A. A higher-order laminated plate theory with improved in-plane response. *International Journal of Solid and Structures*, 1987, 23(1): 111-131

146 Carrera E. On the use of the Murakami's zig-zag function in the modeling of layered plates and shells. *Computers and Structures*, 2004, 82(7-8): 541-554

147 Bhaskar B, Varadan TK. A higher-order theory for bending analysis of laminated shells of revolution. *Computers and Structures*, 1991, 40(3): 815-819

148 Vidal P, Polit O. A sine finite element using a zig-zag function for the analysis of laminated composite beams. *Composites: Part B-Engineering*, 2011, 42(6): 1671-1682

149 Ferreira AJM, Roque CMC, Carrera E, et al. Radial basis functions collocation and a unified formulation for bending, vibration and buckling analysis of laminated plates, according to a variation of Murakami's zig-zag theory. *European Journal of Mechanics - A/Solids*, 2011, 30(3): 559-570

150 Rodrigues JD, Roque CMC, Ferreira AJM, et al. Radial basis functions-finite differences collocation and a unified formulation for bending, vibration and buckling analysis of laminated plates, according to Murakami's zig-zag theory. *Composite Structures*, 2011, 93(5): 1613-1620

151 Rao KM, Meyer-Piening HR. Analysis of thick laminated anisotropic composites plates by the finite element method. *Composite Structures*, 1990, 15(3): 185-213

152 Brank B, Carrera E. Multilayered shell finite element with interlaminar continuous shear stresses: a refinement of the Reissner–Mindlin formulation. *International Journal of Numerical Methods in Engineering*, 2000, 48(6): 843-874

153 Carrera E, Parisch H. An evaluation of geometrical nonlinear effects of thin and moderately thick multilayered composite shells. *Com-*

posite Structures, 1998, 40(1): 11-24

154 Auricchio F, Sacco E. Refined first-order shear deformation theory models for composite laminates. *Journal of Applied Mechanics*, 2003, 70(3): 381-390

155 Carrera E, Demasi L. Multilayered finite plate element based on Reissner mixed variational theorem, part I: theory and part II: numerical analysis. *International Journal for Numerical Methods in Engineering*, 2002, 55(2): 191-231

156 Di Sciuva M. An improved shear-deformation theory for moderately thick multilayered anisotropic shells and plates. *Journal of Applied Mechanics*, 1987, 54(3): 589-596

157 Di Sciuva M. Bending, vibration and buckling of simply supported thick multilayered orthotropic plates: an evaluation of a new displacement model. *Journal of Sound and Vibration*, 1986, 105(3): 425-442

158 Tessler A, Di Sciuva M, Gherlone M. A refined zigzag beam theory for composite and sandwich beams. *Journal of Composite Materials*, 2009, 43(7): 1051-1081

159 Cho M, Parmerter PR. An efficient higher-order plate theory for laminated composites. *Composite Structures*, 1992, 20(2): 113-123

160 Cho M, Parmerter PR. Efficient higher order plate theory for general lamination configurations. *AIAA Journal*, 1993, 31(7): 1299-1308

161 Cho M, Kim KO, Kim MH. Efficient higher-order shell theory for laminated composites. *Composite Structures*, 1996, 34(34): 197-212

162 Ali JSM, Bhaskar K, Varadan TK. A new theory for accurate thermal/ mechanical flexural analysis of symmetric laminated plates. *Composite Structures*, 1999, 45(3): 227-232

163 Cho M, Oh J. Higher order zig-zag plate theory under thermo-electric- mechanical loads combined. *Composites: Part B-Engineering*, 2003, 34(1): 67-82

164 Oh J, Cho M, Kim JS. Buckling analysis of a composite shell with multiple delaminations based on a higher order zig-zag theory. *Finite Elements in Analysis and Design*, 2008, 44(11): 675-685

165 Nguyen SN, Lee J, Cho M. Efficient higher-order zig-zag theory for viscoelastic laminated composite plates. *International Journal of Solids and Structures*, 2015, 62: 174-185

166 Azharia F, Boroomanda B, Shahbazib M. Exponential basis functions in the solution of laminated plates using a higher-order Zig–Zag theory. *Composite Structures*, 2013, 105(8): 398-407

167 Icardi U. A three-dimensional zig-zag theory for analysis of thick laminated beam. *Composite Structures*, 2001, 52(1): 123-135

168 Icardi U. Higher-order zig-zag model for analysis of thick composite beams with inclusion of transverse normal stress and sublaminates approximations. *Composites: Part B–Engineering*, 2001, 32(3): 343-354

169 何陵辉, 刘人怀. 一种考虑层间位移和横向剪应力连续条件的层合板理论. 固体力学学报, 1994, 15(3): 319-326 (He Linghui, Liu Renhuai. A theory for laminated plates considering continuity conditions of interlayer displacements and transverse shear stress. *Acta Mechanica Solida Sinica*, 1994, 15(3): 319-326 (in Chinese))

170 舒小平. 满足板面载荷和横向剪应力连续的层板理论. 复合材料学报, 1994, 11(3): 61-68 (Shu Xiaoping. A laminates and theory accounting for loads on the surfaces and continue of interface shear stresses. *Acta Materiae Compositae Sinica*, 1994, 11(3): 61-68 (in Chinese))

171 舒小平. 复合材料层合板壳热弹性动力响应. 复合材料学报, 1997, 14(2): 103-107 (Shu Xiaoping. Thermomechanical dynamic response of laminated composite plates and shells. *Acta Materiae Compositae Sinica*, 1997, 14(2): 103-107 (in Chinese))

172 白瑞祥, 陈浩然. 含界面脱粘及表板基体开裂损伤的复合材料夹层板非线性稳定性的研究. 复合材料学报, 2002, 19(2): 80-84 (Bai Ruixiang, Chen Haoran. Investigation of nonlinear buckling behavior of composite sandwich plates with interfacial delamination and matrix micro-cracks. *Acta Materiae Compositae Sinica*, 2002, 19(2): 80-84 (in Chinese))

173 任晓辉, 陈万吉. C^0 型高阶锯齿理论及复合材料厚梁热应力分析. 计算力学学报, 2015, 32(1): 77-82 (Ren Xiaohui, Chen Wanji. Thermal stress analysis of thick composite beams based on C^0-type higher-order zig-zag theory. *Chinese Journal of Computational Mechanics*, 2015, 32(1): 77-82 (in Chinese))

174 Oh J, Cho M, Kim JS. Dynamic analysis of composite plate with multiple delaminations based on higher-order zigzag theory. *International Journal of Solids and Structures*, 2005, 42(23): 6122-6140

175 Cho M, Choi YJ. A new postprocessing method for laminated composites of general lamination fonfigurations. *Composite Structures*, 2001, 54(4): 397-406

176 Chalak HD, Chakrabarti A, Iqbal MA, et al. Free vibration analysis of laminated soft core sandwich plates. *Journal of Vibration and Acoustics-Transactions of the ASME*, 2013, 135(135): 011013

177 Zhen W, Lo SH, Sze KY, et al. A higher order finite element including transverse normal strain for linear elastic composite plates with general lamination configurations. *Finite Elements in Analysis and Design*, 2012, 48(1): 1346-1357

178 Xiaohui R, Wanji C, Zhen W. A C^0-type zig-zag theory and finite element for laminated composite and sandwich plates with general configurations. *Archive of Applied Mechanics*, 2012, 82(2): 391-406

179 Reddy JN. A generalization of two-dimensional theories of laminated composite plates. *Communications in Applied Numerical Methods*, 1987, 3(3): 173-180

180 Reddy JN, Barbero EJ, Teply JL. A plate bending element based on a generalized laminated plate theory. *International Journal for Numerical Methods in Engineering*, 1989, 28(10): 2275-2292

181 Barbero EJ, Reddy JN, Teply JL. General two-dimensional theory of laminated cylindrical shells. *AIAA Journal*, 1990, 28(2): 544-553

182 Barbero EJ, Reddy JN. Modeling of delamination in composite laminates using a layer-wise plate theory. *International Journal of Solids and Structures*, 1991, 28(2): 373-388

183 Di Sciuva M, Gherlone M. A global/local third-order Hermitian displacement field with damaged interfaces and transverse extensibility: FEM formulation. *Composite Structures*, 2003, 59(3): 433-444

184 Gherlone M, Di Sciuva M. Thermo-mechanics of undamaged and damaged multilayered composite plates: a sub-laminates finite element approach. *Composite Structures*, 2007, 81(1): 125-136

185 Gherlone M, Di Sciuva M. Thermo-mechanics of undamaged and damaged multilayered composite plates: assessment of the FEM sublaminates approach. *Composite Structures*, 2007, 81(1): 137-155

186 Pai PF, Palazotto AN. A high-order sandwich plate theory accounting for 3-D stresses. *International Journal of Solids and Structures*, 2001, 38(30-31): 5045-5062

187 Carrera E. Mixed layer-wise models for multilayered plates analysis. *Composite Structures*, 1998, 43(1): 57-70

188 Carrera E. Evaluation of layerwise mixed theories for laminated plates analysis. *AIAA Journal*, 1998, 36(4): 830-839

189 Carrera E. Theories and finite elements for multilayered plates and shells. *Archives of Computational Methods in Engineering*, 2003, 10(2): 215-296

190 Carrera E, Demasi L. Two benchmarks to assess two-dimensional theories of sandwich composite plates. *AIAA Journal*, 2003, 41(5): 1356-1362

191 Icardi U. Layerwise mixed element with sublaminates approximation and 3D zig-zag field, for analysis of local effects in laminated and sandwich composites. *International Journal for Numerical Methods in Engineering*, 2007, 70(1): 94-125

192 Marjanovic' M, Vuksanovic' DJ. Layerwise solution of free vibrations and buckling of laminated composite and sandwich plates with embedded delaminations. *Composite Structures*, 2014, 108(1): 9-20

193 Alnefaie K. Finite element modeling of composite plates with internal delamination. *Composite Structures*, 2009, 90(1): 21-27

194 Basar Y, Itskov M, Eckstein A. Composite laminates: nonlinear interlaminar stress analysis by multi-layer shell elements. *Computer Methods in Applied Mechanics and Engineering*, 2000, 185(2-4): 367-397

195 Basar Y, Omurtag MH. Free-vibration analysis of thin/thick laminated structures by layer-wise shell models. *Computers and Structures*, 2000, 74(3): 409-427

196 Botello S, Onate E, Canet JM. A layer-wise triangle for analysis of laminated composite plates and shells. *Computers and Structures*, 1999, 70(6): 635-646

197 Ghoshal A, Kim HS, Chattopadhyay A, et al. Effect of delamination on transient history of smart composite plates. *Finite Elements in Analysis and Design*, 2005, 41(9-10): 850-874

198 Chattopadhyay A, Kim HS, Ghoshal A. Non-linear vibration analysis of smart composite structures with discrete delamination using a refined layerwise theory. *Journal of Sound and Vibration*, 2004, 273(1-2): 387-407

199 胡明勇, 王安稳, 姜伟等. 复合材料层合板的动力响应和横向应力分析. 船舶力学, 2008, 12(4): 778-784 (Hu Mingyong, Wang Anwen, Jiang Wei, et al. Dynamic response and transverse stresses analysis of composite laminated plates. *Journal of Ship Mechanics*, 2008, 12(4): 778-784 (in Chinese))

200 胡明勇，王安稳. 纤维增强粘弹性复合材料层合板的自由振动和应力分析. 工程力学, 2010, 27(6): 10-20 (Hu Mingyong, Wang Anwen. Free vibration and stresses analysis of fiber-reinforced viscoelastic composite laminated plates. *Engineering Mechanics*, 2010, 27(6): 10-20 (in Chinese))

201 赵龙胜, 吴锦武, 赵飞等. 复合材料简支板固有频率与振型分析. 南昌航空大学学报: 自然科学版, 2013, 27(2):10-17 (Zhao Longsheng, Wu Jinwu, Zhao Fei, et al. Natural frequency and modal shape analysis of the simply supported composite plates. *Journal of Nanchang Hangkong University: Natural Sciences*, 2013, 27(2): 10-17 (in Chinese))

202 杨少红, 王安稳. 粘弹层合板的稳态振动和层间应力. 应用力学学报, 2003, 20(3): 112-117 (Yang Shaohong, Wang Anwen. The steady state vibration and interlaminar stresses of viscoelastic laminated plates. *Chinese Journal of Applied Mechanics*, 2003, 20(3): 112-117 (in Chinese))

203 Pandey S, Pradyumna S. A new C^0 higher-order layerwise finite element formulation for the analysis of laminated and sandwich plates. *Composite Structures*, 2015, 131(1): 1-16

204 Plagianakos TS, Saravanos DA. Higher-order layerwise laminate theory for the prediction of interlaminar shear stresses in thick composite and sandwich composite plates. *Composite Structures*, 2009, 87(1): 23-35

205 Reddy JN, Robbins DH. Theories and computational models of composite laminates. *Applied Mechanics Reviews*, 1994, 47 (6): 21-35

206 Noor AK, Burton WS, Bert CW. Computational models for sandwich panels and shells. *Applied Mechanics Reviews*, 1996, 49 (2): 155-199

207 Kant MT. A critical review and some results of recently developed refined theories of fiber-reinforced laminated composites and sandwiches. *Composite Structures*, 1993, 23 (3): 293-312

208 Altenbach H. Theories for laminated and sandwich plates. *Mechanics of Composite Materials*, 1998, 34 (2): 243-252

209 Zhang YX, Yang CH. Recent developments in finite element analysis for laminated composite plates. *Composite Structures*, 2009, 88 (1): 147-157

210 Kreja I. A literature review on computational models for laminated composite and sandwich panels. *Central European Journal of Engineering*, 2011, 1 (1): 59-80

211 Khandan R, Noroozi S, Sewell P, et al. The development of laminated composite plate theories: a review. *Journal of Materials Science*, 2012, 47 (16): 5901-5910

212 Liu RH, Su W. Nonlinear stability of symmetrically laminated cylindrically orthotropic truncated shallow conical shells including transverse shear. *International Journal of Applied Mechanics and Engineering*, 2009, 14(2): 769-790

213 安娜, 王璠, 刘人怀. 复合材料层合圆柱正交异性扁球壳在线布载荷作用下的非线性稳定性. 暨南大学学报 (自然科学版), 2006, 27(2): 398-405 (An Na, Wang Fan, Liu Renhuai. Nonlinear bending of symmetrically laminated cylindrically orthotropic shallow spherical shells under linear load. *Journal of Jinan University (Natural Science)*, 2006, 27(2): 398-405 (in Chinese))

214 丁磊，刘人怀，王璠. 复合材料浅球壳模态分析与结构优化设计. 材料科学与工程学报，2007, 25(2): 439-442 (Ding Lei, Liu Renhuai, Wang Fan. Finite element analysis on vibration modes and structure optimal design of the symmetrically laminated orthotropic shallow shells. *Journal of Materials Science & Engineering*, 2007, 25(2): 439-442 (in Chinese))

215 杨增涛, 徐加初, 王璠. 复合材料层合扁锥壳的冲击屈曲. 振动工程学报, 2007, 20(1): 19-23 (Yang Zengtao, Xu Jiachu, Wang Fan. Impact buckling of laminated composite shallow conical shells. *Journal of Vibration Engineering*, 2007, 20(1): 19-23 (in Chinese))

216 徐加初，杨增涛. 复合材料层合开顶扁球壳在集中冲击下的非线

性动力屈曲. 振动与冲击，2007, 26(1): 1-4 (Xu Jiachu, Yang Zengtao. Dynamic buckling of laminated composite truncated, shallow spherical shells under a concentrated impact. *Journal of Vibration and Shock*, 2007, 26(1): 1-4 (in Chinese))

217 徐加初，杨增涛. 爆炸冲击下复合材料层合扁球壳的动力屈曲. 爆炸与冲击，2007, 2(2): 116-120 (Xu Jiachu, Yang Zengtao. Dynamic buckling of laminated composite shallow spherical shells subjected to explosive impact. *Explosion and Shock Waves*, 2007, 2(2): 116-120 (in Chinese))

218 王璠. 开孔复合材料扁球壳非线性动力响应. 哈尔滨工业大学学报，2000, 32(1): 37-44 (Wang Fan. Nonlinear dynamic response of symmetrically laminated cylindrically orthotropic truncated shallow shells. *Journal of Harbin Institute of Technology*, 2000, 32(1): 37-44 (in Chinese))

219 王志伟，姚著. 蜂窝纸板冲击压缩的试验研究和有限元分析. 机械工程学报, 2012, 48(14): 49-55 (Wang Zhiwei, Yao Zhu. Experimental investigation and finite element analysis for impact compression of honeycomb paperboards. *Journal of Mechanical Engineering*, 2012, 48(14): 49-55 (in Chinese))

220 Wang DM，Wang ZW. Experimental investigation into the cushioning properties of honeycomb paperboard. *Packaging Technology and Science*, 2008, 21(6): 309-316

221 王志伟，王立军，徐晨翼. 多次低强度冲击对蜂窝纸板缓冲性能的影响. 应用力学学报，2015, 32(2): 441-445 (Wang Zhiwei, Wang Lijun, Xu Chenyi. Effects of low-strength repeated impacts on cushioning performance of honeycomb paperboard. *Chinese Journal of Applied Mechanics*, 2015, 32(2): 441-445 (in Chinese))

222 Song ZH, Wang ZH, Ma HW, et al. Mechanical behavior and failure mode of woven carbon/epoxy laminate composites under dynamic compressive loading. *Composites: Part B–Engineering*, 2014, 60(3): 531-536

223 Fan H, Zhao L, Chen H, et al. Ductile deformation mechanisms and designing instructions for integrated woven textile sandwich composites. *Composite Science and Technology*, 2012, 72(9): 1338-1343

224 Fan H, Zhao L, Chen H, et al. Dynamic compression failure mechanisms and dynamic effects of integrated woven sandwich composites. *Journal of Composite Materials*, 2014, 48(3): 427-443

225 何陵辉，成振强，刘人怀. 含非均匀界面相碳/碳纤维复合材料等效热传导性质的研究. 应用数学和力学，1997, 18(2): 193-201 (He Linghui, Cheng Zhenqiang, Liu Renhuai. Investigation on the effective conductivities of carbon/carbon fiber composites with inhomogeneous interphase. *Applied Mathematics and Mechanics*, 1997, 18(2): 193-201 (in Chinese))

226 何陵辉，刘人怀. 球面各向同性颗粒复合材料膨胀系数的界限. 应用数学和力学，1997, 18(3): 317-323 (He Linghui, Liu Renhuai. Bounds of the expansion coefficients of composites reinforced by spherically isotropic particles. *Applied Mathematics and Mechanics*, 1997, 18(3): 317-323 (in Chinese))

227 何陵辉，成振强，刘人怀. 空心球复合材料热弹性性质的一些精确结果. 应用数学和力学，1998, 19(4): 399-406 (He Linghui, Cheng Zhenqiang, Liu Renhuai. Some exact results concerning thermoelastic properties of hollow sphere composites. *Applied Mathematics and Mechanics*, 1998, 19(4): 399-406 (in Chinese))

228 王璠，何一帆，宋毅等. 引发方式、铺层对纤维增强复合材料圆柱壳吸能特性影响的冲击试验研究. 振动工程学报, 2013, 26(1): 33-40 (Wang fan, He Yifan, Song Yi, et al. The impact experimental study of evoking type and Layup type on absorption energy of fiber reinforced composite cylindrical shell. *Journal of Vibration Engineering*, 2013, 26(1): 33-40 (in Chinese))

229 郑伟玲，肖潭，朱朦琪等. 聚苯乙炔包覆多壁碳纳米管的制备及其分散性. 物理化学学报, 2009, 25(11)：2373-2379 (Zheng Weiling, Xiao Tan, Zhu Mengqi, et al. Preparation and dispersivity of multiwalled carbon nanotubes coated by poly (phenylacetylene). *Acta Physico-Chimica Sinica*, 2009, 25(11)：2373-2379 (in Chinese))

230 卫保娟，肖潭，李雄俊等. 石墨烯与多壁碳纳米管增强环氧树脂复合材料的制备及性能. 复合材料学报, 2012, 29(4): 53-59 (Wei Baojuan, Xiao Tan, Li Xiongjun, et al. Preparation and properties of graphene and MWCNTs reinforced epoxy resin composites. *Acta Materiae Compositae Sinica*, 2012, 29(4): 53-59 (in Chinese))

231 朱朦琪，肖潭，郑伟玲等. 混杂功能化碳纳米管/聚氨酯复合材料的制备及性能. 材料研究学报, 2012, 26(2):191-198 (Zhu Mengqi, Xiao Tan, Zheng Weiling, et al. Preparation and properties of hybrid functionalized carbon nanotube polyurethane composites. *Chinese Journal of Materials Research*, 2012, 26(2):191-198 (in Chinese))

232 姜久红，王志伟. 蜂窝型缓冲包装系统的振动特性分析. 湖北工业大学学报, 2006, 21(2): 18-20 (Jiang Jiuhong, Wang Zhiwei. The research on the vibration Characteristics for honeycomb paperboard cushioning package system. *Journal of Hubei University of Technology*, 2006, 21(2): 18-20 (in Chinese))

233 Wang ZW, E YP. Energy-absorbing properties of paper honeycombs under low and intermediate strain rates. *Packaging Technology and Science*, 2012, 25(2): 173-185

234 宁志华，刘人怀，刘稀南. 径向约束下含非均匀界面层纤维增强复合材料的热弹性分析// 第十三届现代数学和力学学术会议暨钱伟长诞辰 100 周年纪念大会论文集, 上海，2012-10-6–2012-10-8 (Ning Zhihua, Liu Renhuai, Liu Xinan. Thermal analysis for fiber-reinforced composites containing inhomogeneous interphase subjected to a radial constrain//Preceeding of the Conference of 13th Modern Mathematics and Mechanics (MMM-XIII) & the 100th Anniversary of Weichang Chien's Birth. Shanghai, 6th-8th Oct., 2012-10-6–2012-10-8 (in Chinese))

235 Ning ZH, Liu XN. Thermal analysis for fiber-reinforced composites containing inhomogeneous interphase subject to a radial constraint. *Advanced Materials Research*, 2013, 652-654: 77-83

236 刘述伦，薛江红，王璠. 纤维增强正交各向异性复合材料层合板的湿热屈曲. 暨南大学学报 (自然科学版), 2013, 34(4): 489-494 (Liu Shulun, Xue Jianghong, Wang Fan. Hygrothermal buckling of fiber-reinforced, symmetrical cross-ply composite plates. *Journal of Jinan University (Natural Science)*, 2013, 34(4): 489-494 (in Chinese))

237 刘述伦，薛江红，王璠等. 含脱层纤维增强复合材料层合板在湿热环境下的屈曲分析. 工程力学, 2015, 32(2): 1-8 (Liu Shulun, Xue Jianghong, Wang Fan, et al. Buckling analysis of fiber-reinforced laminated composite plate with delamination under hygrothermal environments. *Engineering Mechanics*, 2015, 32(2): 1-8 (in Chinese))

238 朱永安，王璠. 均布载荷和温度场联合作用下的扁球壳的热屈曲. 南昌大学学报, 2007, 31(增刊): 205-207 (Zhu Yongan, Wang Fan. Thermal buckling of shallow spherical shell under uniformly distributed load and temperature field. *Journal of Nanchang University*, 2007, 31(sup): 205-207 (in Chinese))

239 朱永安，王璠, 刘人怀. 考虑横向剪切的对称圆柱正交异性层合扁球壳的热屈曲. 应用数学和力学, 2008, 29(2): 263-271 (Zhu Yongan, Wang Fan, Liu Renhuai. Thermal buckling of axisymmetrically laminated cylindrically orthotropic shallow spherical shells including transverse shear. *Applied Mathematics and Mechanics*, 2008, 29(2): 263-271 (in Chinese))

240 E YP，Wang ZW. Effect of relative humidity on energy absorption properties of honeycomb paperboards. *Packaging Technology and Science*, 2010, 23(6): 471-483

241 E YP，Wang ZW. Plateau stress of paper honeycomb as response to various relative humidities. *Packaging Technology and Science*, 2010, 23(3): 203-216

242 陈炎，韩景龙，刘人怀. 横观各向同性压电矩形薄板的非线性振动. 南京航空航天大学学报, 2003, 35(1): 18-24 (Chen Yan, Han Jinglong, Liu Renhuai. Nonlinear vibration of transversely isotropic piezoelectric rectangular plate. *Journal of Nanjing University of Aeronautics& Astronautics*, 2003, 35(1): 18-24 (in Chinese))

243 陈炎，刘人怀. 压电矩形薄板的非线性强迫振动. 华南理工大学学报 (自然科学版), 2003, 31(S1): 63-66 (Chen Yan, Liu Renhuai. Nonlinear force vibration of piezoelectric rectangular thin plate. *Journal of South China University of Technology* (*Natural Science Edition*), 2003, 31(S1): 63-66 (in Chinese))

244 陈炎，刘人怀. 单向轴压下压电矩形薄板的动态后屈曲问题. 振动工程学报, 2008, 21(3): 335-342 (Chen Yan, Liu Renhuai. Dynamic post-buckling of piezoelectric rectangular thin plate under unilateral axially-pressure. *Journal of Vibration Engineering*, 2008, 21(3): 335-342 (in Chinese))

245 陈炎，刘人怀. 考虑位移场和电场高阶分布的压电板耦合非线性理论. 刘人怀等主编. 工程力学科学与实践, 广州：暨南大学出版社, 2010: 183-214 (Chen Yan, Liu Renhuai. A Nonlinear Theory for Piezoelectric Plates With Coupling Effects of High Order Displacement and Electric Field. Liu Renhuai et al. Eds. Science and practice in engineering mechanics. Guangzhou: Jinan University Press, 2010: 183-214 (in Chinese))

246 Wang H, Hu Y, Wang J. On the nonlinear behavior of a multilayer circular piezoelectric plate-like transformer operating near resonance. *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*, 2013, 60(3): 752-757

247 Wang H, Hu Y, Wang J. Nonlinear analysis of a 5-layer beam-like piezoelectric transformer near resonance. *Acta Mechanica Solida Sinica*, 2014, 27(2): 195-201

248 傅衣铭, 李升, 姜叶洁. 具界面损伤压电智能层合板的非线性自由振动分析. 应用数学和力学, 2009, 30(2): 127-141 (Fu Yiming, Li Sheng, Jiang Yejie. Non-linear free vibration analysis of piezoelastic laminated plates with interfacial damage. *Applied Mathematics and Mechanics*, 2009, 30(2): 127-141 (in Chinese))

249 郑玉芳, 傅衣铭, 王锋. 具损伤压电层合板的非线性动力稳定性分析. 力学学报, 2006, 38(3): 570-576 (Zheng Yufang, Fu Yiming, Wang Feng. Nonlinear dynamic stability for piezoelectric laminated plates with damage. *Chinese Journal of Theoretical and Applied Mechanics*, 2006, 38(3): 570-576 (in Chinese))

250 Rao MN, Schmidt R, Schröder KU. Geometrically nonlinear static FE-simulation of multilayered magneto-electro-elastic composite structures. *Composite Structures*, 2015, 127: 120-131

251 Xiao Y, Zhou HM, Cui XL. Nonlinear resonant magnetoelectric coupling effect with thermal, stress and magnetic loadings in laminated composites. *Composite Structures*, 2015, 128: 35-41

252 Yu GL, Zhang HW, Li YX, et al. Theoretical study of nonlinear magnetoelectric response in laminated magnetoelectric composites. *Composite Structures*, 2014, 108: 287-294

253 Shi Y, Gao YW. Theoretical study on nonlinear magnetoelectric effect and harmonic distortion behavior in laminated composite. *Journal of Alloys and Compounds*, 2015, 646: 351-359

254 舒小平. 压电复合材料层板柱面弯曲的精化理论. 应用力学学报, 2005, 22(1): 44-48 (Shu Xiaoping. Accurate theory of laminated piezoelectric composite plates under cylindrical bending. *Chinese Journal of Applied Mechanics*, 2005, 22(1): 44-48(in Chinese))

255 Xue JH, Wang YO, Yuan D. A shear deformation theory for bending and buckling of undersea sandwich pipes. *Composite Structures*, 2015, 132: 633-643

256 Xue JH. Asymptotic analysis for buckling of undersea corroded pipelines. *ASME Transaction—Journal of Pressure Vessel Technology*, 2008, 130 (2): 021705

257 Xue JH. A nonlinear finite element analysis of buckle propagation in subsea corroded pipelines. *Finite Elements in Analysis and Design*, 2006, 42(14-15): 1211-1219

258 Xue J, Hoo Fatt MS. Symmetric and anti-symmetric buckle propagation modes in subsea corroded pipelines. *Marine Structures*, 2005, 18(1): 43-61

259 Williams JG, Hadavinia H. Analytical solutions for cohesive zone models. *Journal of the Mechanics and Physics of Solids*, 2002, 50(4): 809-825

260 Blackman BRK, Hadavinia H, Kinloch AJ, et al. The use of a cohesive zone model to study the fracture of fibre composites and adhesivelybonded joints. *International Journal of Fracture*, 2003, 119(1): 25-46

261 Pike MG, Oskay C. XFEM modeling of short microfiber reinforced composites with cohesive interfaces. *Finite Elements in Analysis and Design*, 2015, 106: 16-31

262 Ouyang ZY, Li GQ. Cohesive zone model based analytical solutions for adhesively bonded pipe joints under torsional loading. *International Journal of Solids and Structures*, 2009, 46(5): 1205-1217

263 Lorenzis LD, Zavarise G. Cohesive zone modeling of interfacial stresses in plated beams. *International Journal of Solids and Structures*, 2009, 46(24): 4181-4191

264 Yuan H, Teng JG, Seracino R, et al. Full-range behavior of FRP-to-concrete bonded joints. *Engineering Structure*, 2004, 26(5): 553-565

265 Yuan H, Lu XS, Hui D, et al. Studies on FRP-concrete interface with hardening and softening bond-slip law. *Composite Structures*, 2012, 94(19): 3781-3792

266 Chen JF, Yuan H, Teng JG. Debonding failure along a softening FRP-to-concrete interface between two adjacent cracks in concrete

members. *Engineering Structures*, 2007, 29: 259-270

267 赵玉萍, 袁鸿, 韩军. 基于弹性 – 塑性内聚力模型的纤维拔出界面宏观行为分析. 力学学报, 2015, 47(1): 127-134 (Zhao Yuping, Yuan Hong, Han Jun. Analysis of the macroscopic interfacial behaviour of the fibre pullout using elastic-plastic cohesive model. *Chinese Journal of Theoretical and Applied Mechanics*, 2015, 47(1): 127-134 (in Chinese))

268 刘三星, 袁鸿. FRP 抗剪加固混凝土梁中间裂缝的扩展过程. 暨南大学学报, 2014, 35(1): 104-112 (Liu Sanxing, Yuan Hong. Process of middle crack propagation in concrete beams shear-strengthened with FRP strips. *Journal of Jinan University*, 2014, 35(1): 104-112(in Chinese))

269 关天发, 袁鸿. CFRP 加固钢筋混凝土曲梁的抗弯性能试验研究. 中山大学学报 (自然科学版), 2008, 47(增刊 2): 109-113 (Guan Tianfa, Yuan Hong. Experimental study on bending properties of RC curved beam strengthened with CFRP. *Acta Scientiarum Naturalium Universitatis Sunyatseni*, 2008, 47 (Sup.2): 109-113(in Chinese))

270 袁鸿, 李庆, 萧澎伟等. 侧面黏贴纤维复合材料的抗剪加固梁破坏机理. 暨南大学学报 (自然科学版), 2012, 33(4): 463-467 (Yuan Hong, Li Qing, Xiao Pengwei, et al. Failure mechanism of shear strengthened beam with side-bonded FRP. *Journal of Jinan University (Natural Science)*, 2012, 33(4): 463-467 (in Chinese))

271 向红, 傅衣铭, 洪力. 受面内双向载荷蜂窝材料的弹性屈曲模式及其影响因素. 应用力学学报, 2009, 26(2): 321-324 (Xiang Hong, Fu Yiming, Hong Li. Elastic buckling mode and influence factors of hexagonal honeycombs under biaxial loading. *Chinese Journal of Applied Mechanics*, 2009, 26(2): 321-324 (in Chinese))

272 向红, 傅衣铭, 洪力. 不规则蜂窝材料在单向载荷作用下的蠕变屈曲分析. 固体力学学报, 2008, 29(2): 181-186 (Xiang Hong, Fu Yiming, Hong Li. Creep-buckling of irregular hexagonal honeycombs under uniaxial loading. *Chinese Journal of Solid Mechanics*, 2008, 29(2): 181-186 (in Chinese))

273 Wang YJ, Wang ZM. Creep buckling of cross-ply symmetric laminated cylindrical panels. *Applied Mathematics and Mechanics*, 1993, 14(3): 313-318

274 Xue JH, Luo QZ, Han F, et al. Two dimensional analyses of delamination buckling of symmetrically cross-ply, rectangular laminates. *Applied Mathematics and Mechanics*, 2013, 34(4): 597-612

275 Xue JH, Zi L, Yuan H, et al. Contact analysis for fiber reinforced, delaminated laminates with kinematic nonlinearity. *Acta Mechanica Solida Sinica*, 2013, 26(3): 388-402

276 Tang MQ, Xue JH, Yuan H, et al. Macro-micro analysis for anti-penetrating postbuckling of T300/QY8911 carbon fiber reinforced, deboned laminates with contact effects. *International Journal of Applied Mechanics*, 2014, 6(4): 1450044

277 Zhu F, Fu Y, Chen D. Analysis of fatigue delamination growth for piezoelectric laminated cylindrical shell considering nonlinear contact effect. *International Journal of Solids and Structures*, 2008, 45: 5381-5396

278 杨金花, 傅衣铭. 具脱层复合材料层合圆柱壳的屈曲分析. 湖南大学学报 (自然科学版), 2005, 32(4): 25-30 (Yang Jinhua, Fu Yiming. Buckling analysis of composite laminated cylindrical shell with delamination. *Journal of Hunan University (Natural Sciences)*, 2005, 32(4): 25-30 (in Chinese))

279 杨金花, 傅衣铭. 具脱层轴对称层合圆柱壳的振动模态分析. 应用力学学报，2007, 24(3): 509-513 (Yang Jinhua, Fu Yiming. Modal analysis of axisymmetrical laminated cylindrical shell with delamination. *Chinese Journal of Applied Mechanics*, 2007, 24(3): 509-513 (in Chinese))

280 傅衣铭, 李升, 姜叶洁. 具界面损伤压电智能层合板的非线性自由振动分析. 应用数学和力学, 2009, 30(2): 127-141 (Fu Yiming, Li Sheng, Jiang Yejie. Non-linear free vibration analysis of piezoelastic laminated plates with interfacial damage. *Applied Mathematics and Mechanics*, 2009, 30(2): 127-141 (in Chinese))

281 陈得良，傅衣铭. 考虑接触效应的具脱层轴对称层合圆板的非线性动力响应. 固体力学学报，2007, 28(3): 406-410 (Chen Deliang, Fu Yiming. Nonlinear dynamic response of an axisymmetricly-delaminated circular plate with considering contact effect. *Acta Mechanica Solida Sinic*, 2007, 28(3): 406-410 (in Chinese))

282 Chen HR, Wang M, Bai RX. The effect of nonlinear contact upon natural frequency of delaminated stiffened composite plate. *Composite Structures*, 2006, 76(1-2): 28-33

283 Chen HR, Hong M, Liu YD. Dynamic behavior of delaminated plates considering progressive failure process. *Composite Structures*, 2004, 66(1-4): 456-466

284 Schwarts-Givli H, Rabinovitch O, Frostig Y. High-order nonlinear contact effects in the dynamic behavior of delaminated sandwich panels with a flexible core. *International Journal of Solids and Structures*, 2007, 44(1): 77-99

285 Wang J, Tong L. A study of the vibration of delaminated beams using a nonlinear anti-interpenetration constraint model. *Composite Structures*, 2002, 57(1-4): 483-488

286 Kwon YW, Aygunes H. Dynamic finite element analysis of laminated beams with delamination cracks using contact-impact conditions. *Computers and Structures*, 1996, 58(6): 1161-1169

287 Whitcomb JD. Analysis of a laminate with a postbuckled embedded delamination, including contact effects. *Journal of Composite Materials*, 1992, 26(8): 1523-1535

288 Yeh MK, Fang LB, Kao MH. Bending behavior of delaminated composite plates with contact effects. *Composite Structures*, 1997, 39(3-4): 347-356

289 Yeh MK, Fang LB. Contact analysis and experiment of delaminated cantilever composite beam. *Composites: Part B-Engineering*, 1999, 30(4): 407-414

290 Luo H, Hanagud S. Dynamics of delaminated beams. *International Journal of solid and structures*, 2000, 37(8): 1501-1519

291 Chen H, Bai R. Postbuckling behavior of face/core debonded composite sandwich plate considering matrix crack and contact effect. *Composite Structures*, 2002, 57(1-4): 305-313

292 Hu N, Fukunaga H, Sekine H, et al. Compressive buckling of laminates with an embedded delamination. *Composite Science and Technology*, 1999, 59(8): 1247-1260

293 Bruno D, Greco F, Lonetti P. A coupled interface-multilayer approach for mixed mode delamination and contact analysis in laminated composites. *International Journal of Solids and Structures*, 2003, 40(26): 7245-7268